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Digital Image Processing

2. Digital Image Fundamentals

- **Structure of the Human Eye**
  - **FIGURE 2.1**
  - Figure 2.2 shows the density of rods and cones for a cross section of the right eye passing through the region of emergence of the optic nerve from the eye.
    - by taking some liberty in interpretation, we can view the fovea as a square sensor array of size 1.5 mm*1.5 mm.
    - a charge-coupled device (CCD) imaging chip of medium resolution can have this number of elements in a receptor array no larger than 3 mm*3 mm.

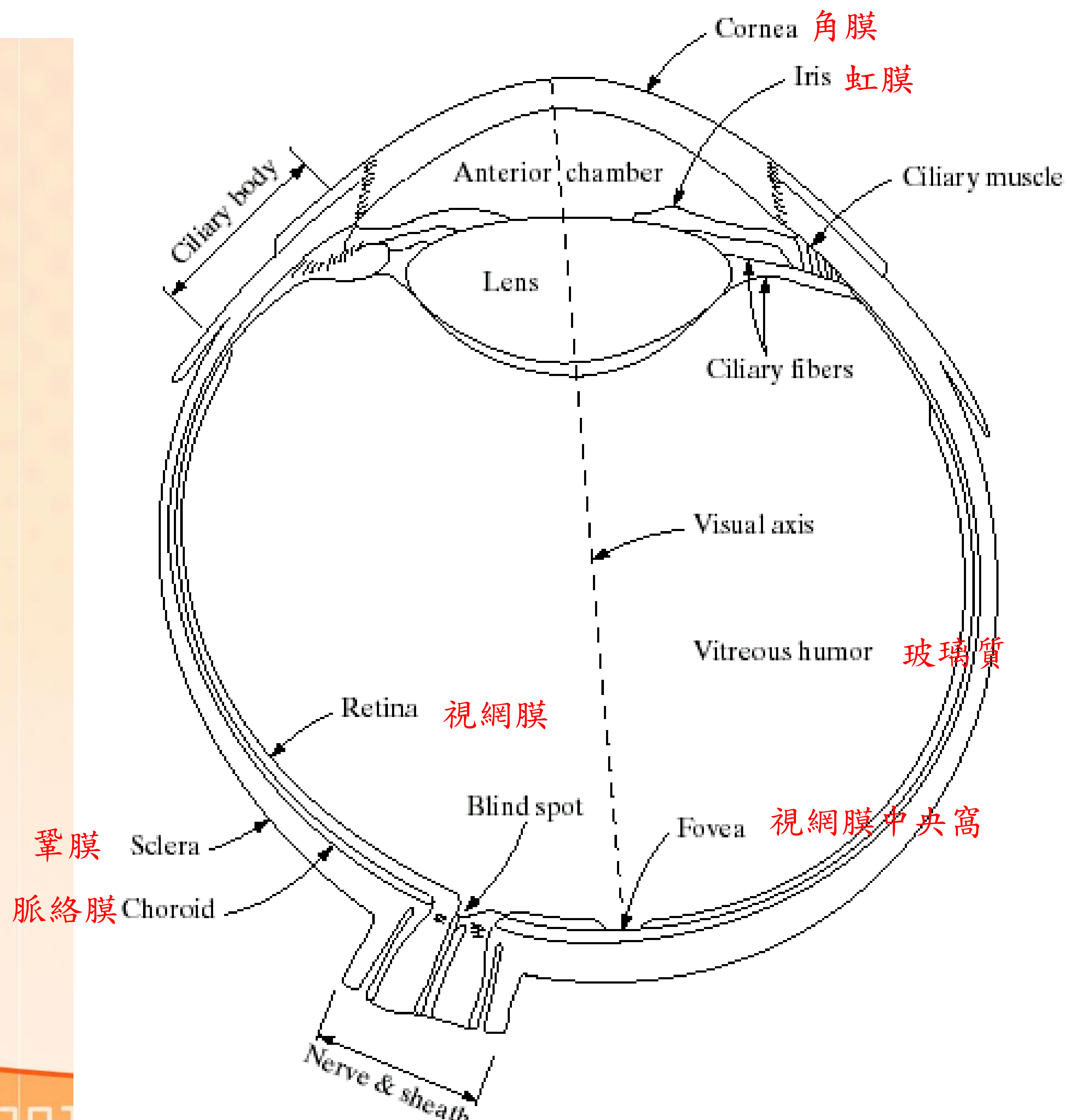


FIGURE 2.1
Simplified diagram of a cross section of the human eye.

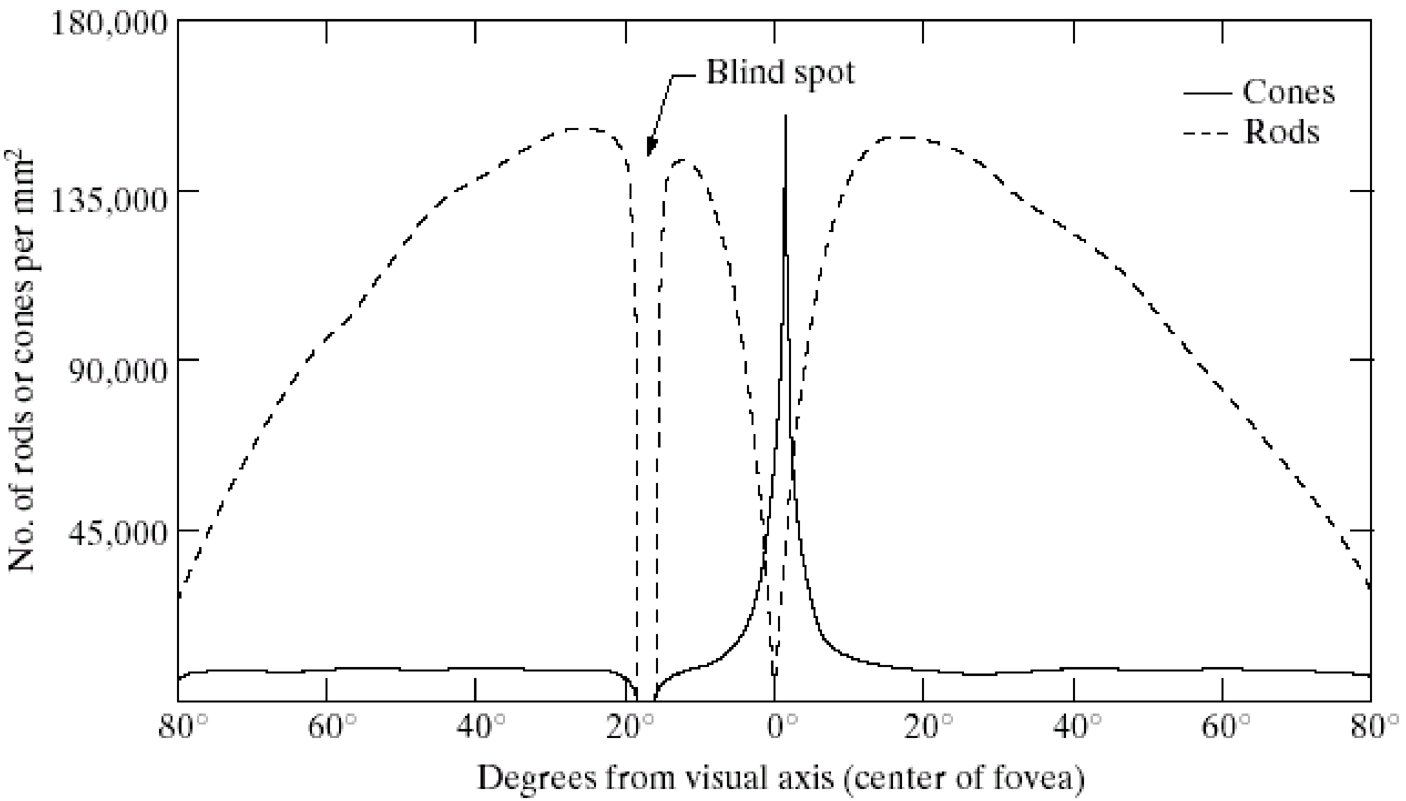
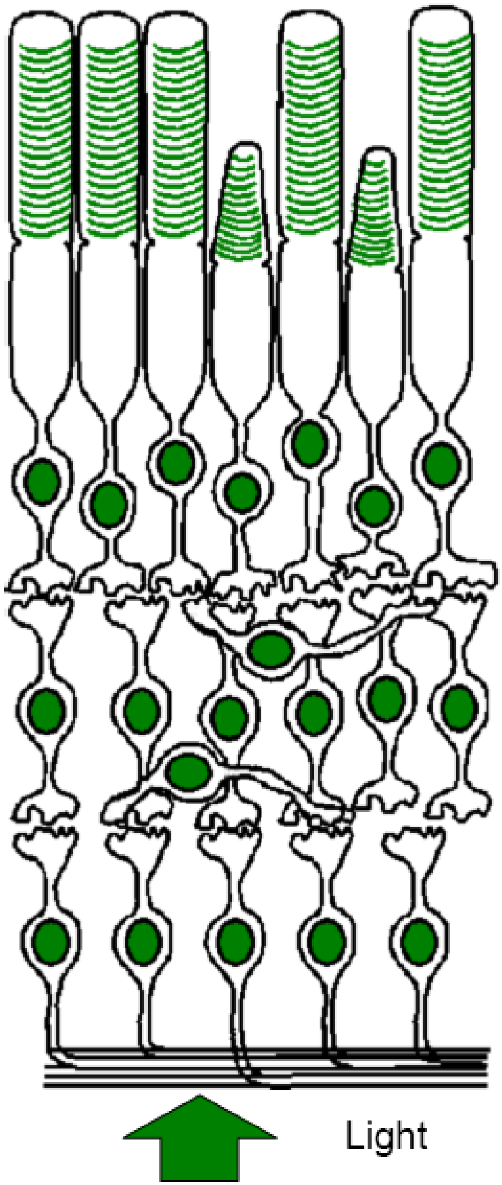
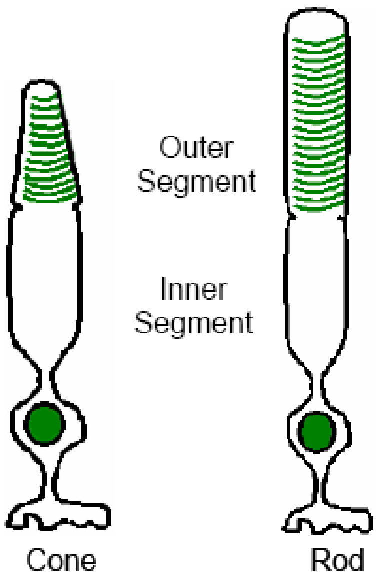


FIGURE 2.2 Distribution of rods and cones in the retina.

Retina





Cones	Rods
Color Vision	Monochromatic
No sensitivity in the dark	High sensitivity in the dark
Respond in bright light	Bleached in bright light
Slow temporal response	Fast temporal response
Mostly in fovea	Mostly in periphery
Some in peripheral retina	None in fovea
High visual acuity	Low visual acuity
In fovea, one neuron per cone	Many rods per single neuron

- **Image Formation in the Eye**
  - In Fig. 2.3
    - the observer is looking at a tree 15 m high at a distance of 100 m. If h is the height in mm of that object in the retinal image, the geometry of Fig. 2.3 yields $15/100=h/17$ or $h=2.55$ mm.
- **Brightness Adaptation and Discrimination**
  - The essential point in interpreting the impressive dynamic range depicted in Fig. 2.4
    - the visual system cannot operate over such a range *simultaneously*.

FIGURE 2.3
Graphical representation of the eye looking at a palm tree. Point *C* is the optical center of the lens.

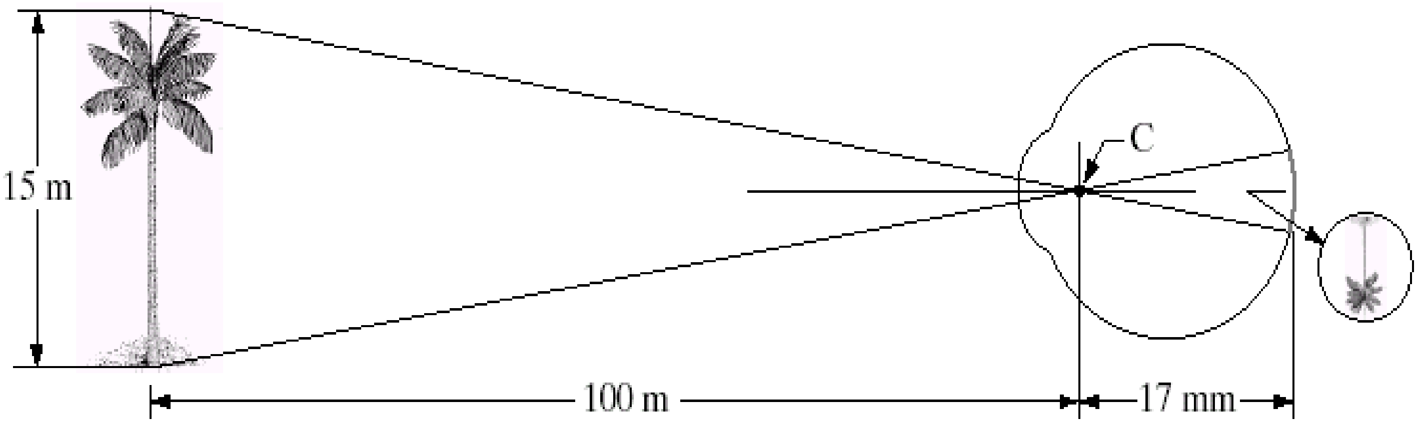
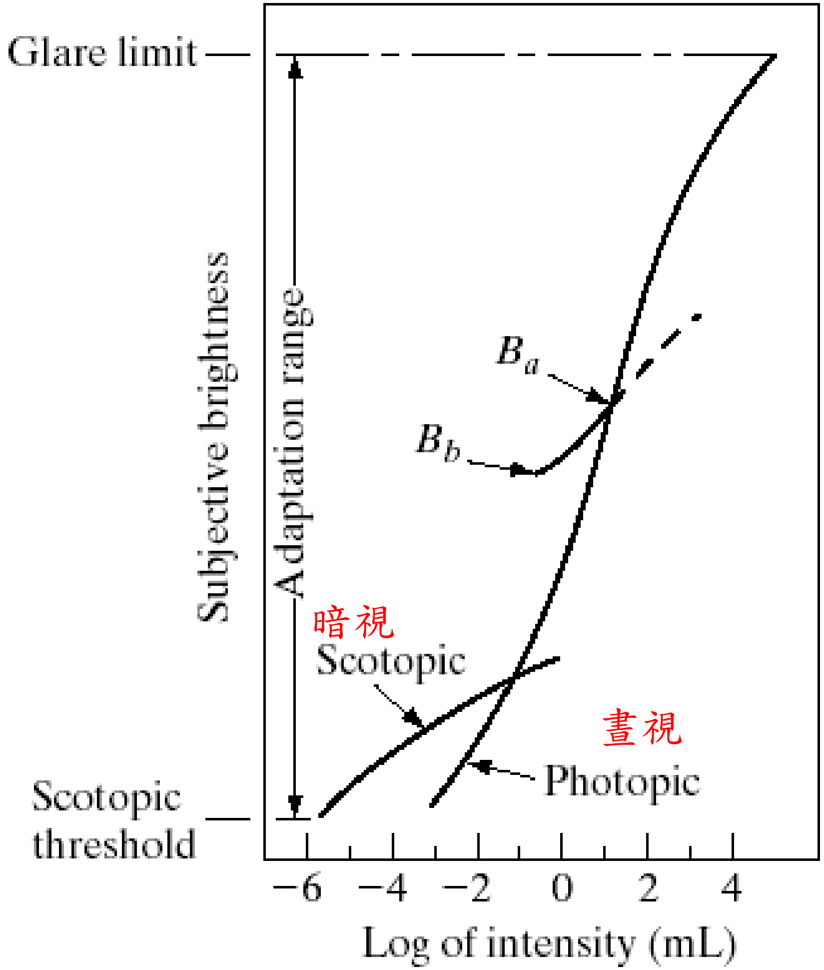


FIGURE 2.4
Range of subjective brightness sensations showing a particular adaptation level.



2.1 Elements of Visual Perception

- A classic experiment used to determine the capability of the human visual system for brightness discrimination consists of having a subject look at a flat, uniformly illuminated area large enough to occupy the entire field of view.
  - This area typically is a diffuser, such as opaque glass, that is illuminated from behind by a light source whose intensity, I, can be varied.
  - To this field is added an increment of illumination, I, in the form of a short-duration flash that appears as a circle in the center of the uniformly illuminated field, as Fig. 2.5 shows.

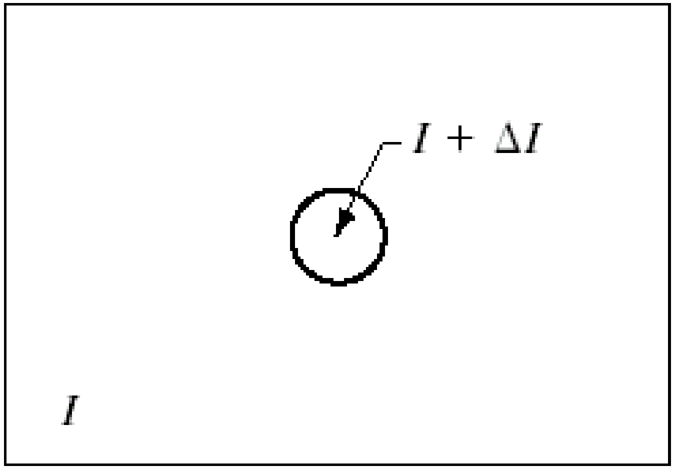


FIGURE 2.5 Basic experimental setup used to characterize brightness discrimination.

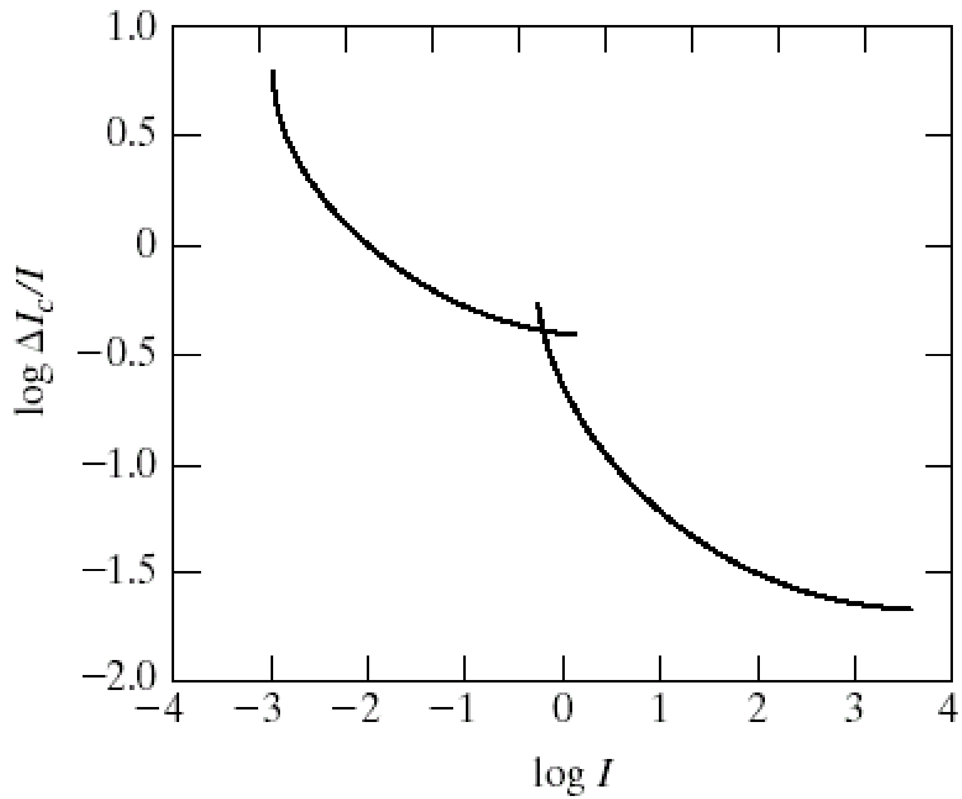
2.1 Elements of Visual Perception

- If ΔI is not bright enough, the subject says “no,” indicating no perceivable change.
- As ΔI gets stronger, the subject may give a positive response of “yes,” indicating a perceived change.
- when ΔI is strong enough, the subject will give a response of “yes” all the time.

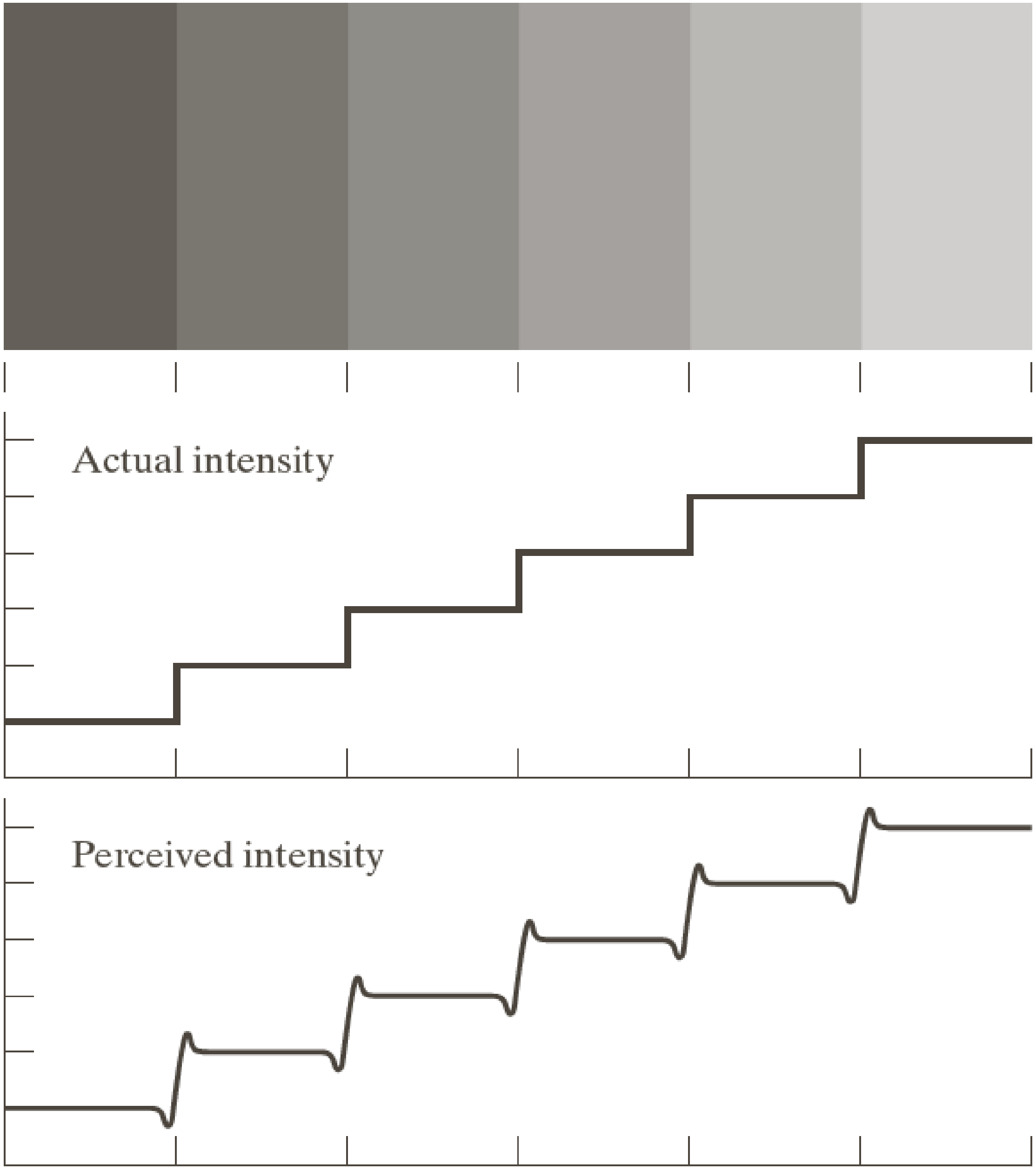
- The quantity $\Delta I_c / I$ where ΔI_c is the increment of illumination discriminable 50% of the time with background illumination I, is called the *Weber ratio*.

- A small value of $\Delta I_c / I$ means that a small percentage change in intensity is discriminable.
  - This represents "good" brightness discrimination.
- A large value of $\Delta I_c / I$ means that a large percentage change in intensity is required.
  - This represents "poor" brightness discrimination.
- A plot of as a function of $log I$ has the general shape shown in Fig. 2.6.

FIGURE 2.6
Typical Weber ratio as a function of intensity.

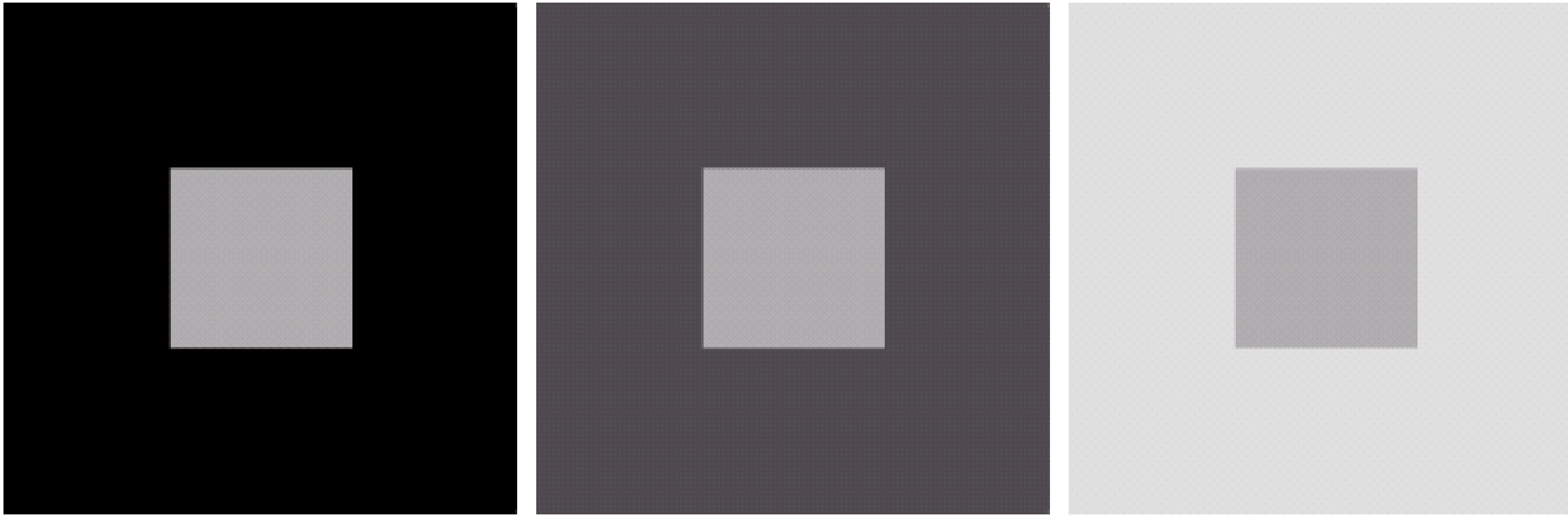


- In fact, the eye objectionable contouring effects in monochrome images whose overall intensity is represented by fewer than approximately two dozen levels.
  - Figure 2.7 shows a striking example of this phenomenon.
  - Fig. 2.8 demonstrates *simultaneous contrast*.
- Optical illusions
  - Some examples are shown in Fig. 2.9.



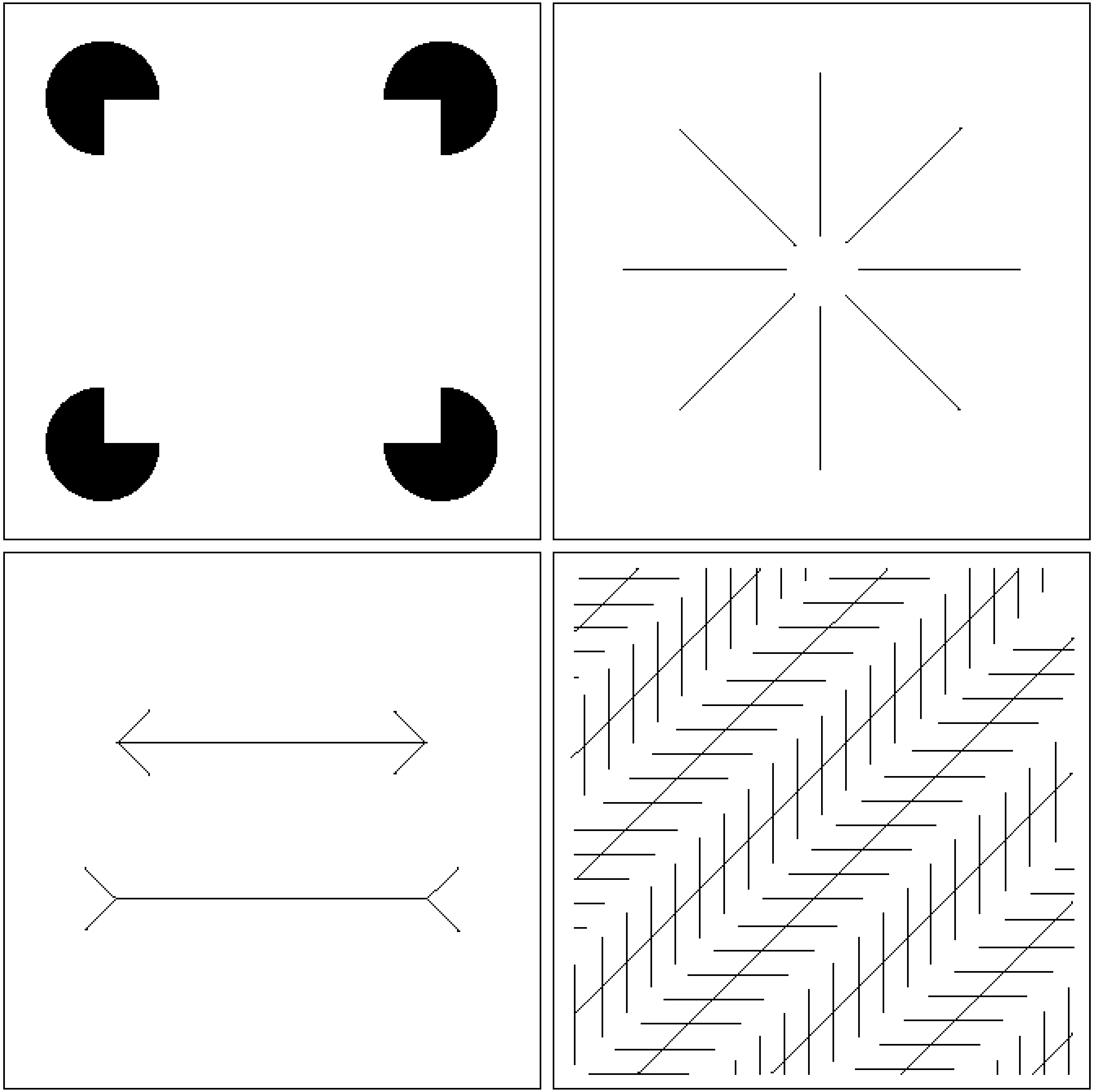
a
b
c

FIGURE 2.7 Illustration of the Mach band effect. Perceived intensity is not a simple function of actual intensity.



a b c

FIGURE 2.8 Examples of simultaneous contrast. All the inner squares have the same intensity, but they appear progressively darker as the background becomes lighter.



a b
c d

FIGURE 2.9 Some well-known optical illusions.

- The range of colors we perceive in visible light represents a very small portion of the electromagnetic spectrum.
  - As shown in Fig. 2.10.
  - Wavelength (λ) and frequency (v) are related by the expression

$\lambda = c / v$ (2.2-1) where c is the speed of light ($2.998*10^8$ m/s).

  - The energy of the various components of the electromagnetic spectrum is given by the expression

$E = hv$ (2.2-2) where *h* is Planck's constant.The units of wavelength are meters, with the terms *microns* (denoted m and equal to 10^{-6} m) and *nanometers* (10^{-9} m) being used just as frequently. Frequency is measured in Hertz (Hz), with one Hertz being equal to one cycle of a sinusoidal wave per second.

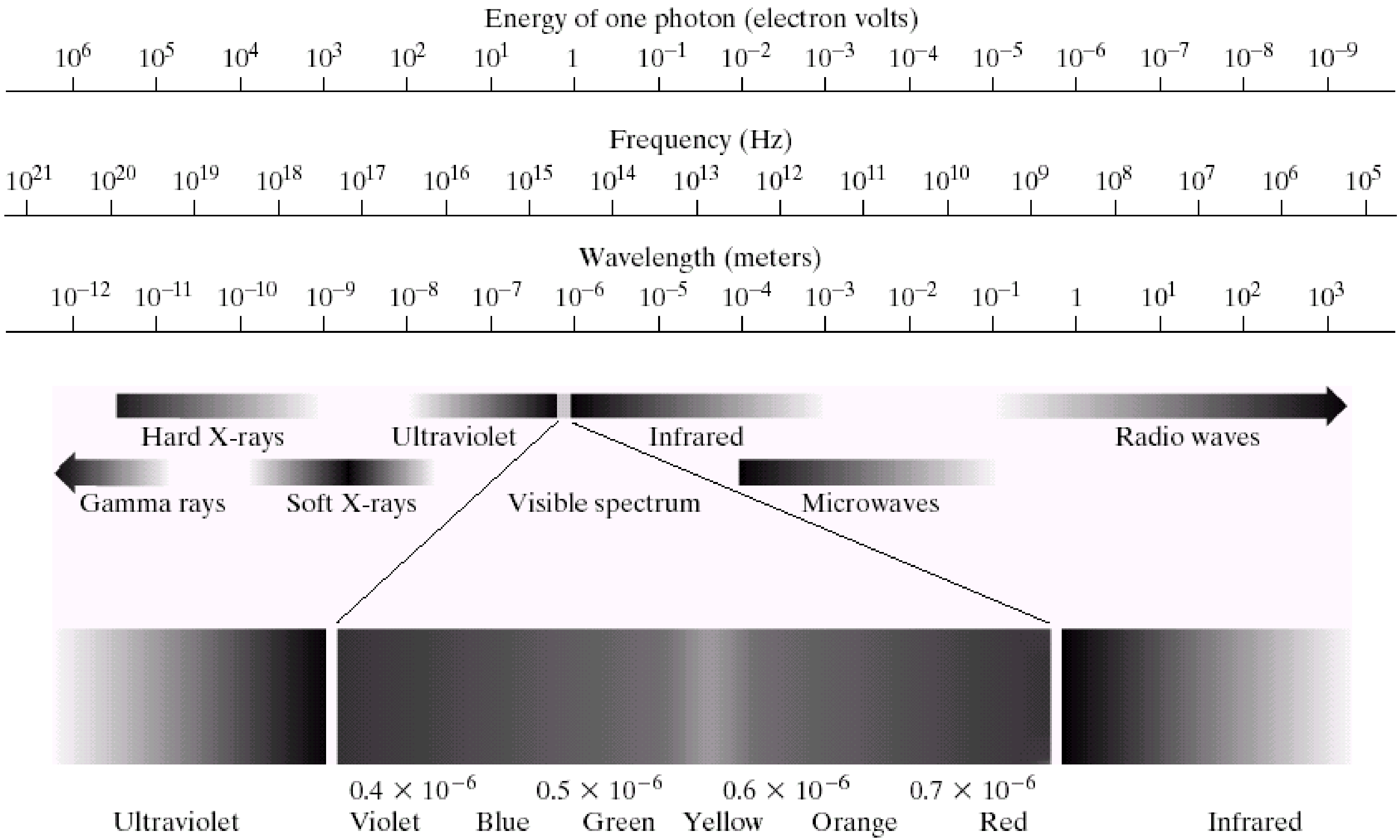
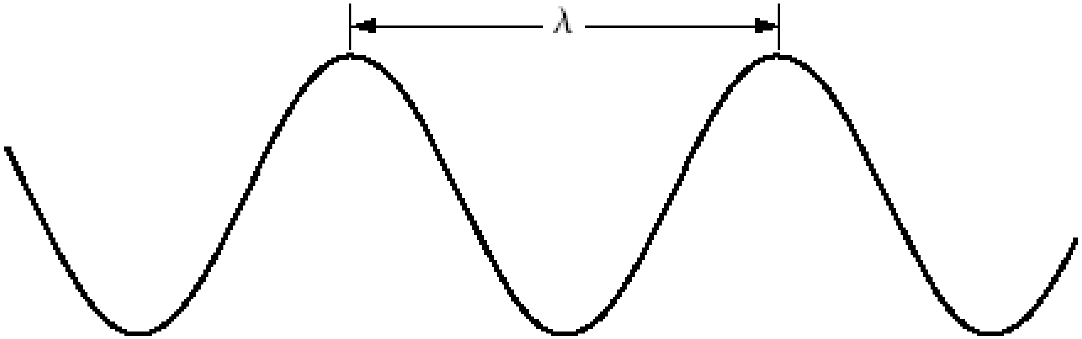


FIGURE 2.10 The electromagnetic spectrum. The visible spectrum is shown zoomed to facilitate explanation, but note that the visible spectrum is a rather narrow portion of the EM spectrum.

FIGURE 2.11
Graphical representation of one wavelength.



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- Figure 2.12 shows the three principal sensor arrangements used to transform illumination energy into digital images.
  - **Single Sensor**
    - Figure 2.13 shows an arrangement used in high-precision scanning.
    - A film negative is mounted onto a drum whose mechanical rotation provides displacement in one dimension.
  - **Sensor Strips**
    - Sensor strips mounted in a ring configuration are used in medical and industrial imaging to obtain cross-sectional ("slice") images of 3-D objects
    - Fig. 2.14.
  - **Sensor Arrays**
    - **FIGURE 2.15 :** An example of the digital image acquisition process.

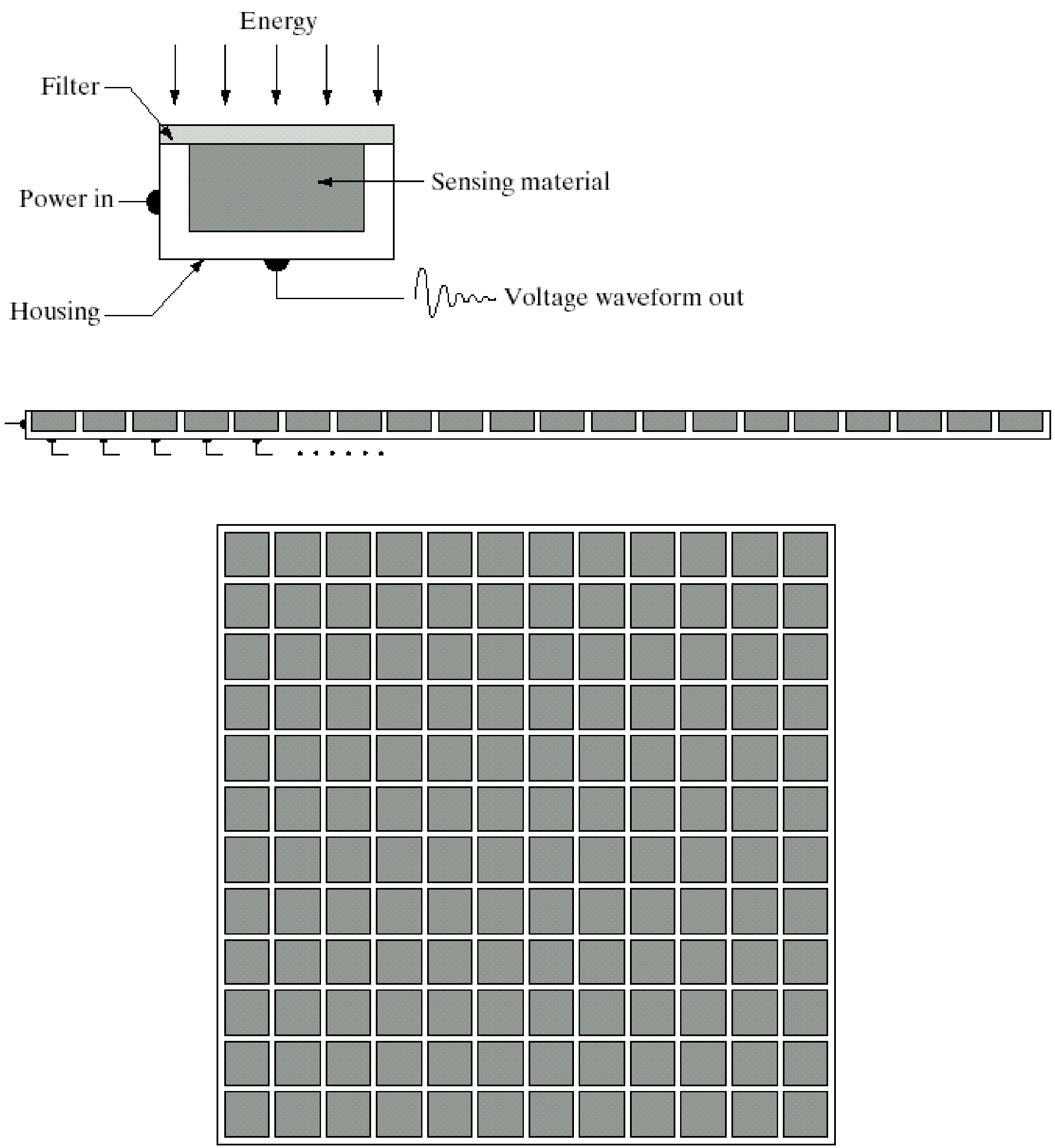


FIGURE 2.12
(a) Single imaging sensor.
(b) Line sensor.
(c) Array sensor.

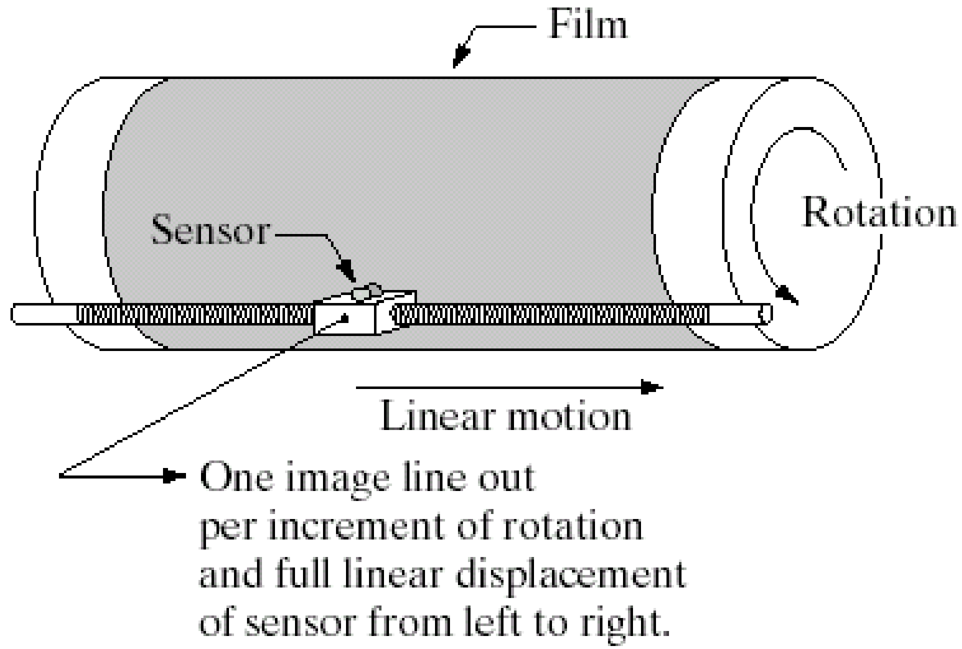
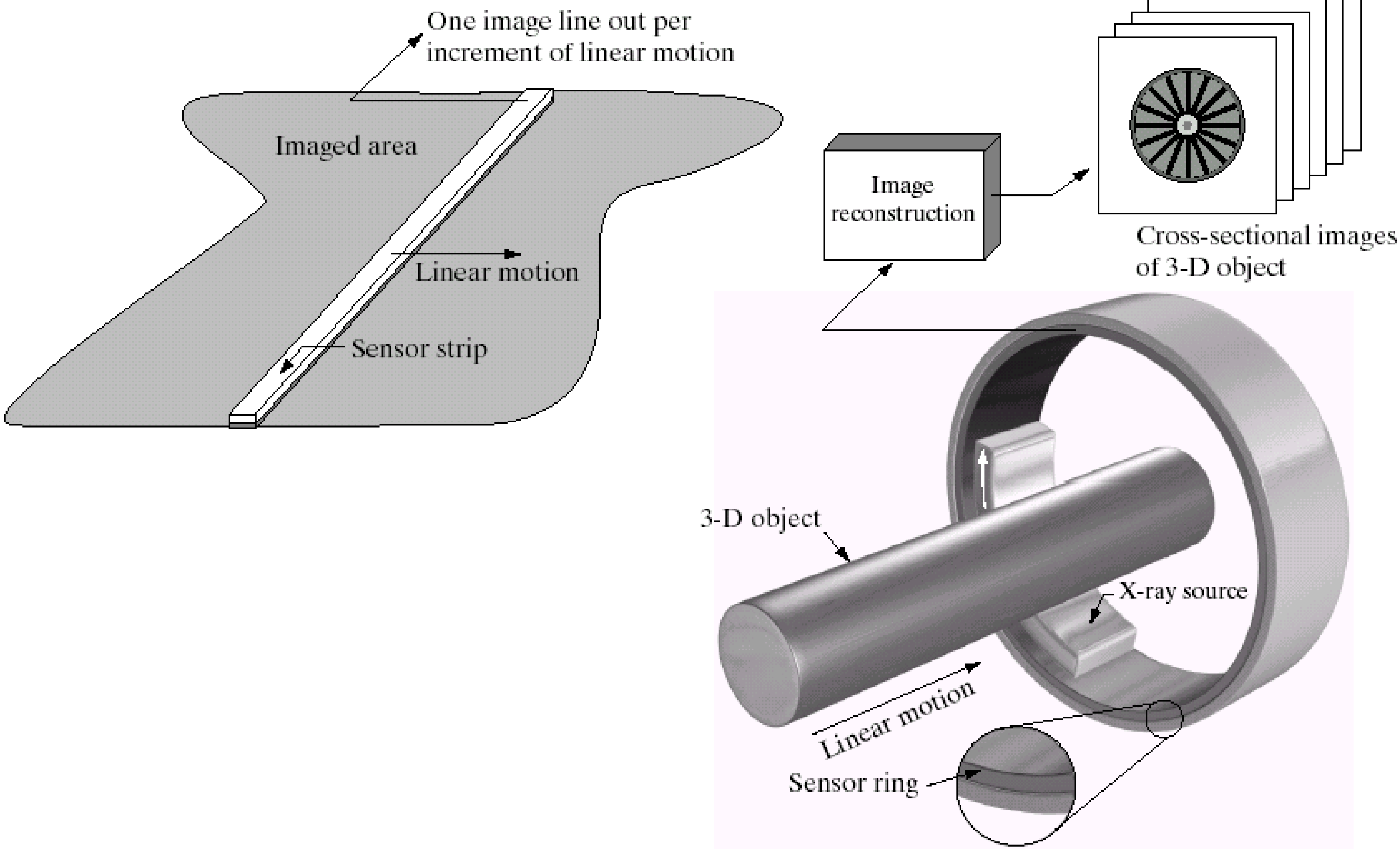


FIGURE 2.13 Combining a single sensor with motion to generate a 2-D image.



a b

FIGURE 2.14 (a) Image acquisition using a linear sensor strip. (b) Image acquisition using a circular sensor strip.

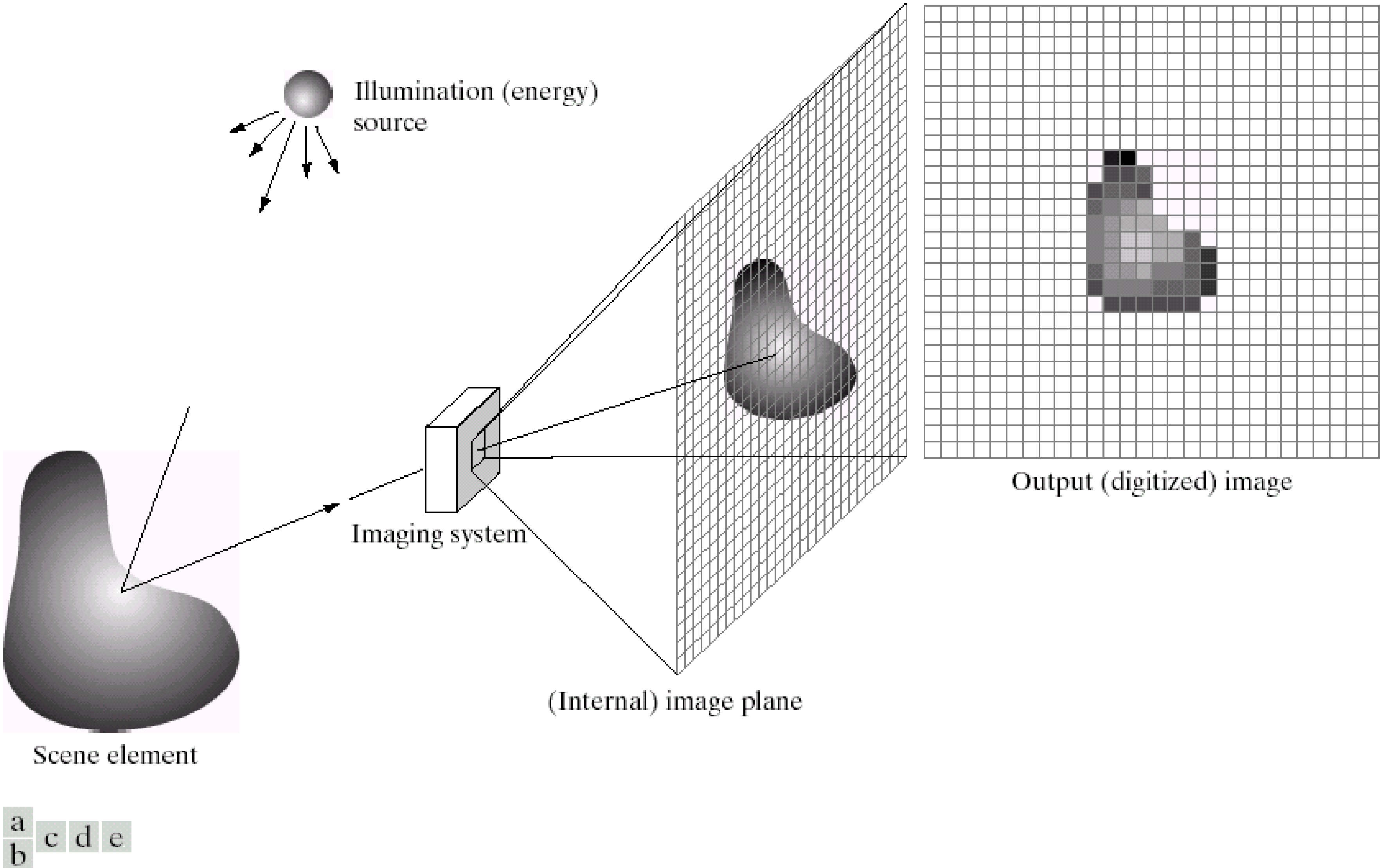


FIGURE 2.15 An example of the digital image acquisition process. (a) Energy ("illumination") source. (b) An element of a scene. (c) Imaging system. (d) Projection of the scene onto the image plane. (e) Digitized image.

2.4 Image Sampling and Quantization

- Digitizing the coordinate values is called *sampling*.
- Digitizing the amplitude values is called *quantization*.
  - Figure 2.16
  - Figure 2.17

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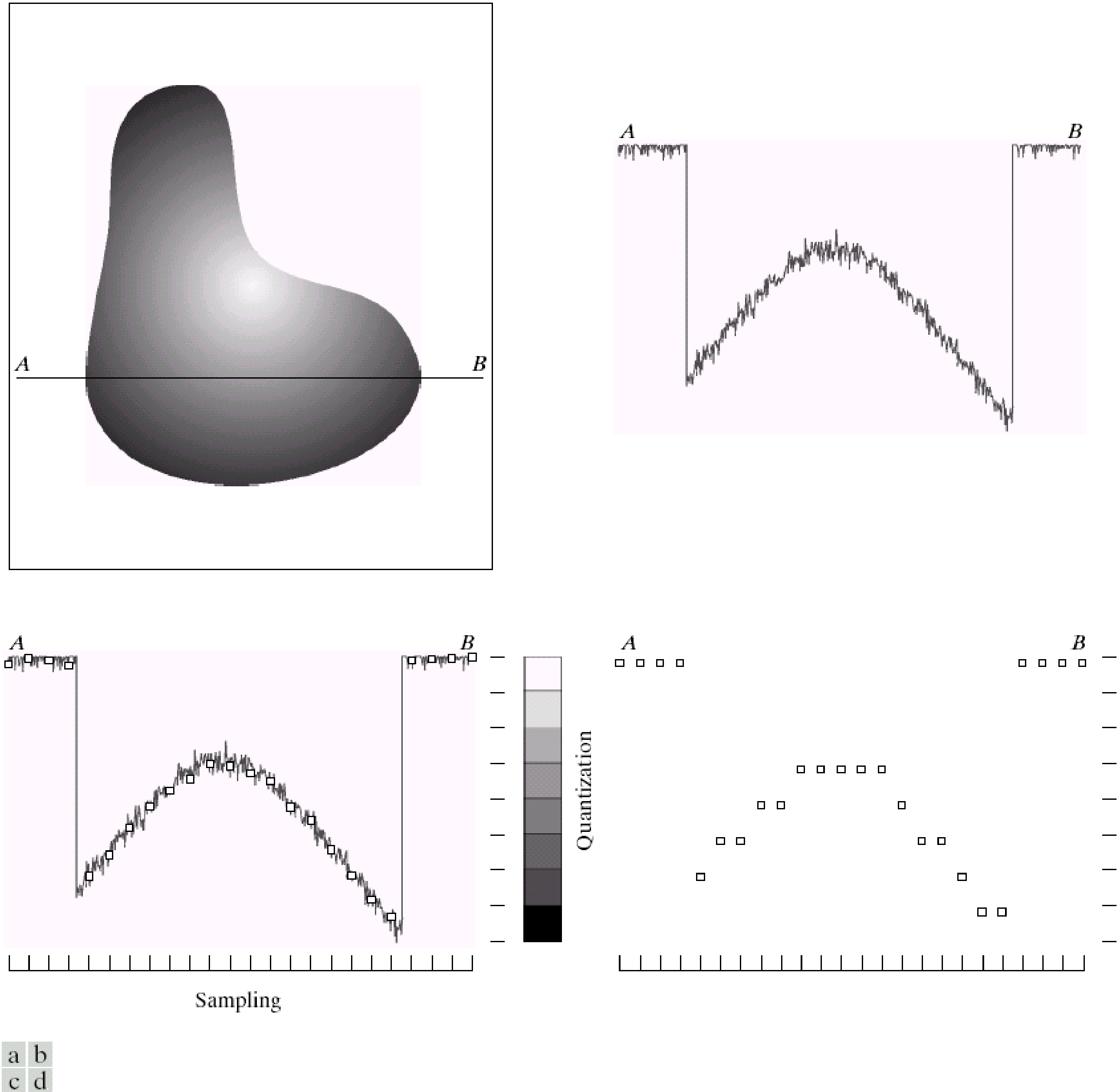
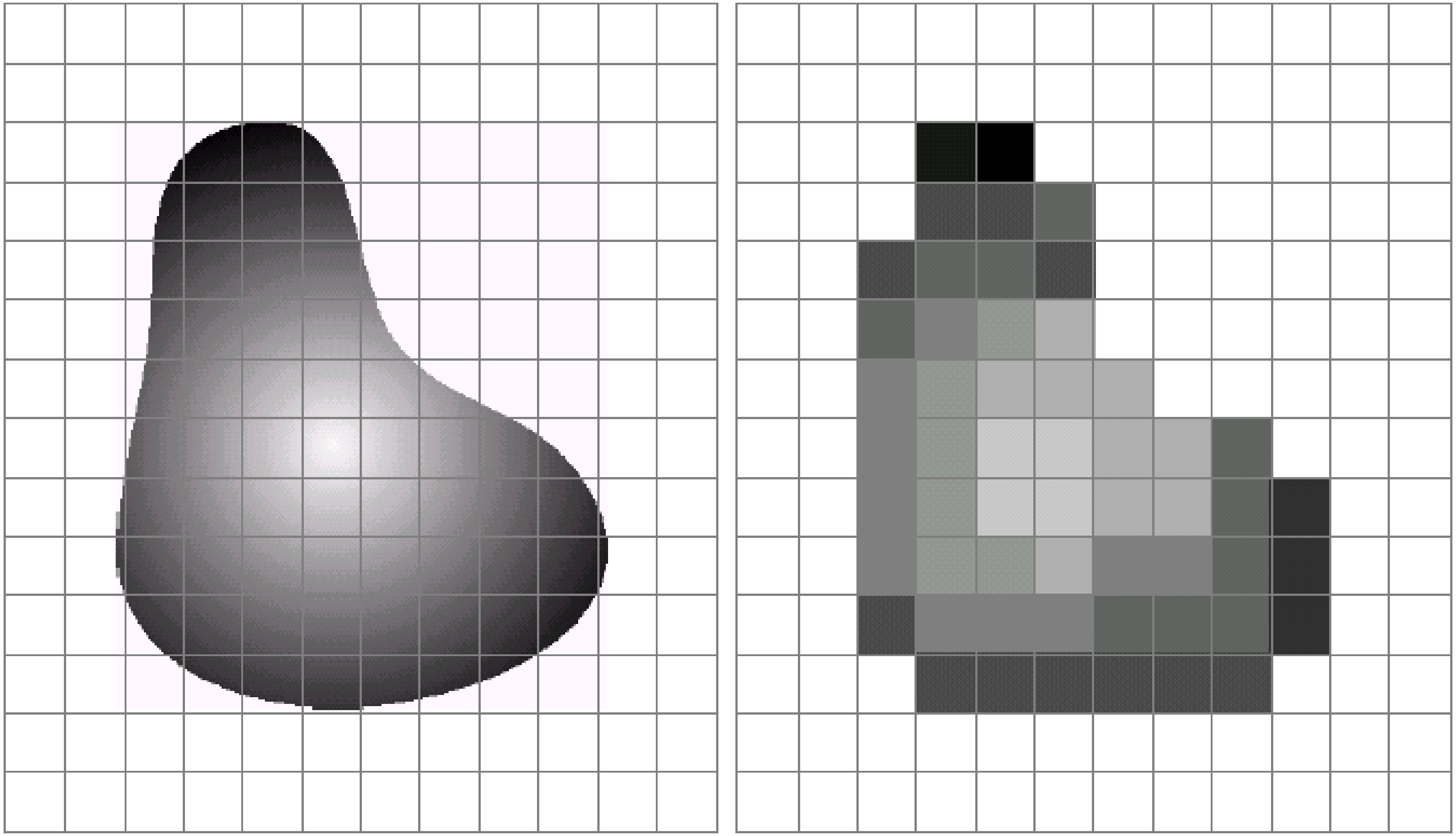
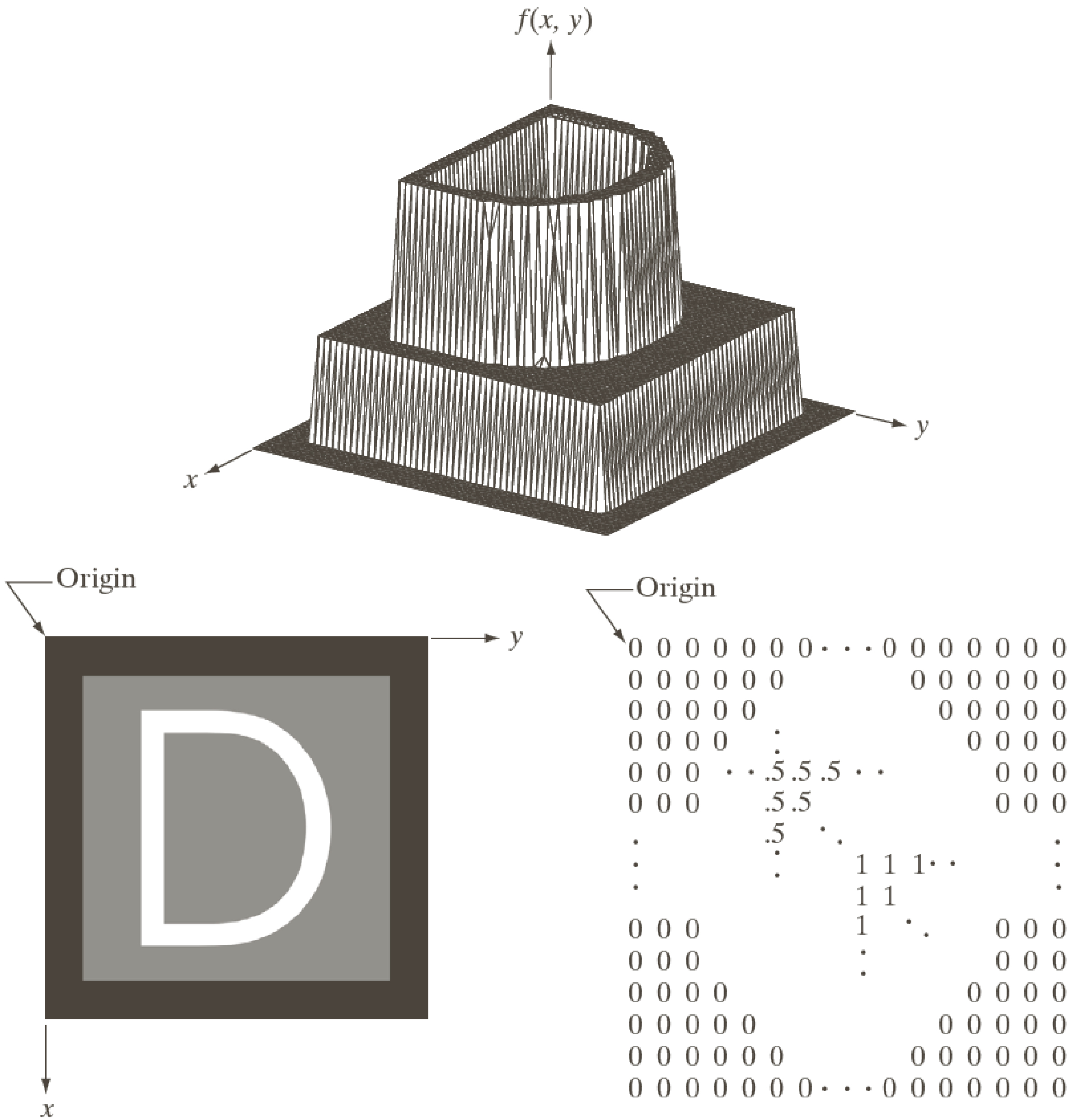


FIGURE 2.16 Generating a digital image. (a) Continuous image. (b) A scan line from A to B in the continuous image, used to illustrate the concepts of sampling and quantization. (c) Sampling and quantization. (d) Digital scan line.



a b

FIGURE 2.17 (a) Continuos image projected onto a sensor array. (b) Result of image sampling and quantization.



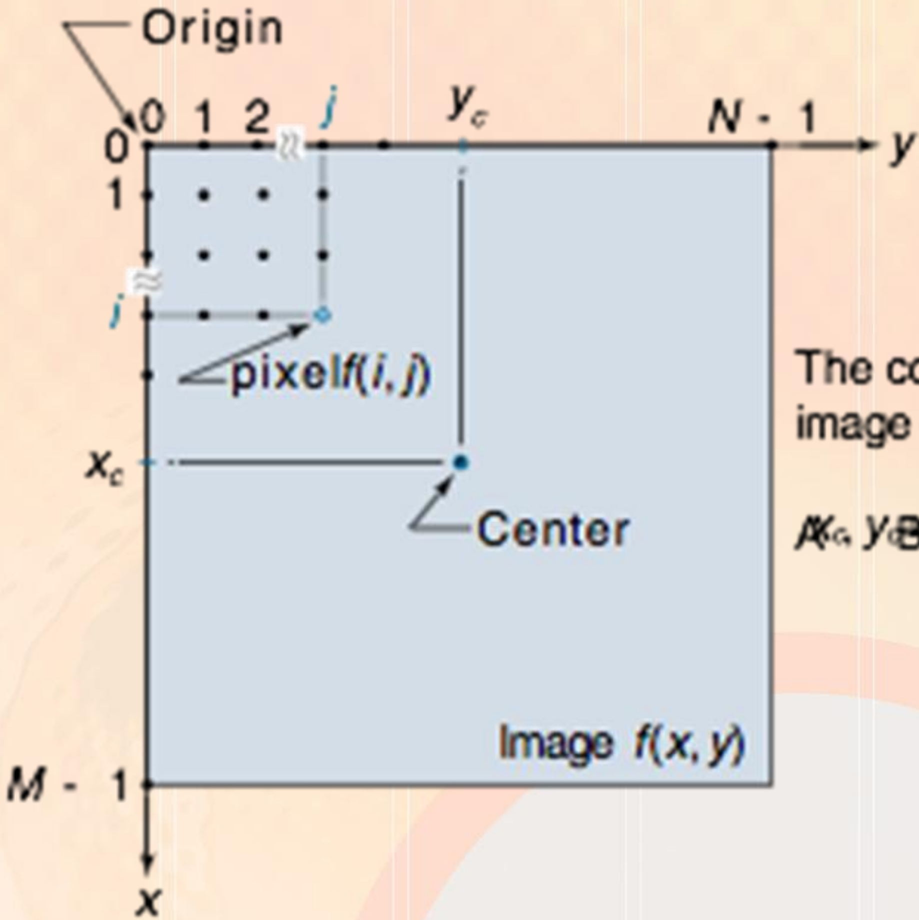
a
b c

FIGURE 2.18
(a) Image plotted as a surface.
(b) Image displayed as a visual intensity array.
(c) Image shown as a 2-D numerical array (0, .5, and 1 represent black, gray, and white, respectively).

- **Representing Digital Images**
  - Figure 2.18
  - The notation introduced in the preceding paragraph allows us to write the complete $M \times N$ digital image in the following compact matrix.
    - This digitization process requires decisions about values for M, N, and for the number, L, of discrete gray levels allowed for each pixel.
    - There are no requirements on *M* and *N*, other than that they have to be positive integers.
    - Due to processing, storage, and sampling hardware considerations, the number of gray levels typically is an integer power of 2:

$$L = 2^k.$$

FIGURE 2.19 Coordinate convention used to represent digital images. Because coordinate values are integers, there is a one-to-one correspondence between x and y and the rows (r) and columns (c) of a matrix.



The coordinates of the image center are

$$(x_c, y_c) = \left(\text{floor}\left(\frac{M}{2}\right), \text{floor}\left(\frac{N}{2}\right)\right)$$

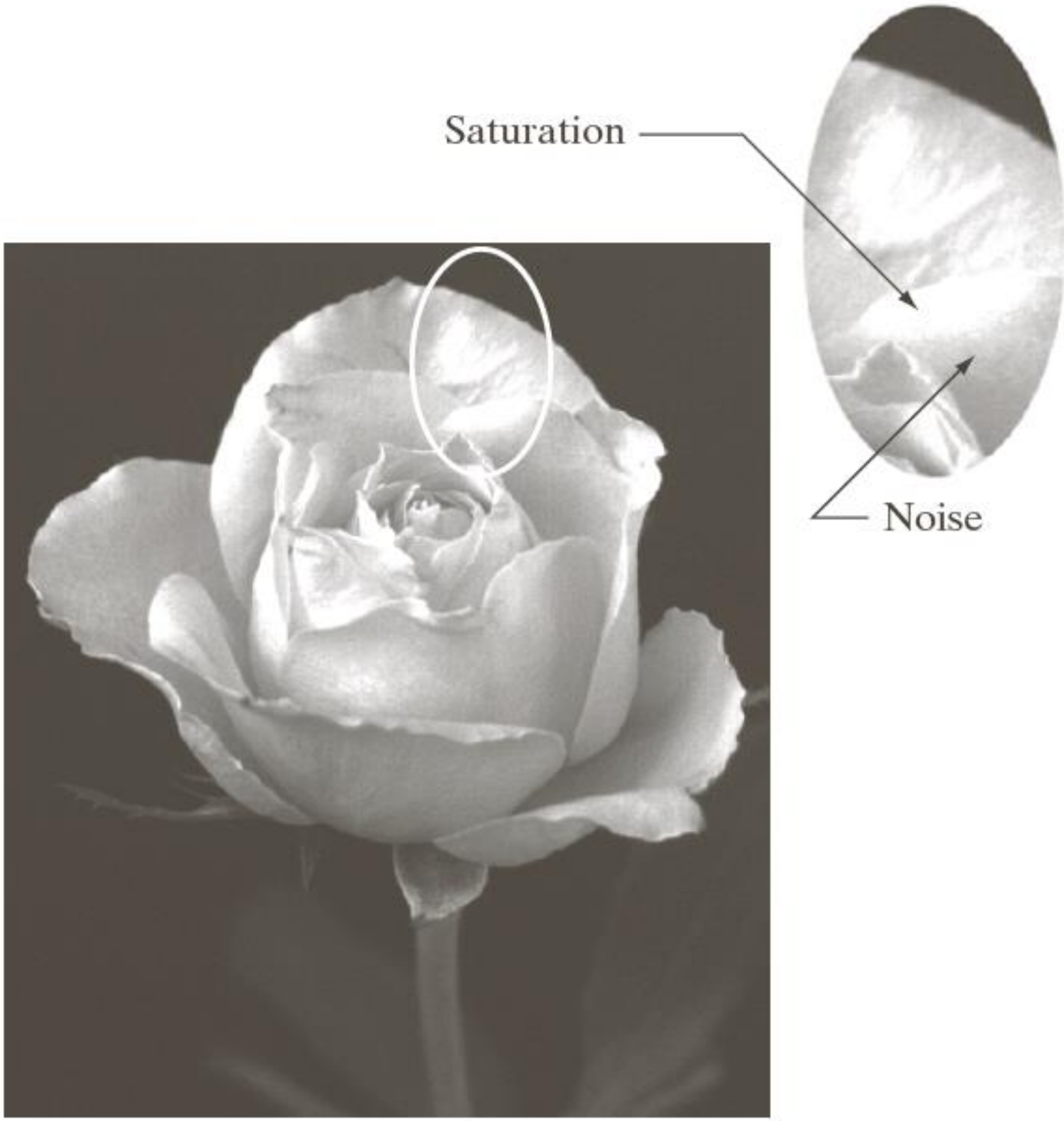


FIGURE 2.19 An image exhibiting saturation and noise. Saturation is the highest value beyond which all intensity levels are clipped (note how the entire saturated area has a high, *constant* intensity level). Noise in this case appears as a grainy texture pattern. Noise, especially in the darker regions of an image (e.g., the stem of the rose) masks the lowest detectable true intensity level.

- The number, b, of bits required to store a digitized image is

$$b = M \times N \times k. \quad (2.4\text{-}4)$$

- When $M=N$, this equation becomes

$$b = N^2 k. \quad (2.4\text{-}5)$$

- Table 2.1 shows the number of bits required to store square images with various values of N and k.

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TABLE 2.1
Number of storage bits for various values of N and k.

N/k	1 (L = 2)	2 (L = 4)	3 (L = 8)	4 (L = 16)	5 (L = 32)	6 (L = 64)	7 (L = 128)	8 (L = 256)
32	1,024	2,048	3,072	4,096	5,120	6,144	7,168	8,192
64	4,096	8,192	12,288	16,384	20,480	24,576	28,672	32,768
128	16,384	32,768	49,152	65,536	81,920	98,304	114,688	131,072
256	65,536	131,072	196,608	262,144	327,680	393,216	458,752	524,288
512	262,144	524,288	786,432	1,048,576	1,310,720	1,572,864	1,835,008	2,097,152
1024	1,048,576	2,097,152	3,145,728	4,194,304	5,242,880	6,291,456	7,340,032	8,388,608
2048	4,194,304	8,388,608	12,582,912	16,777,216	20,971,520	25,165,824	29,369,128	33,554,432
4096	16,777,216	33,554,432	50,331,648	67,108,864	83,886,080	100,663,296	117,440,512	134,217,728
8192	67,108,864	134,217,728	201,326,592	268,435,456	335,544,320	402,653,184	469,762,048	536,870,912

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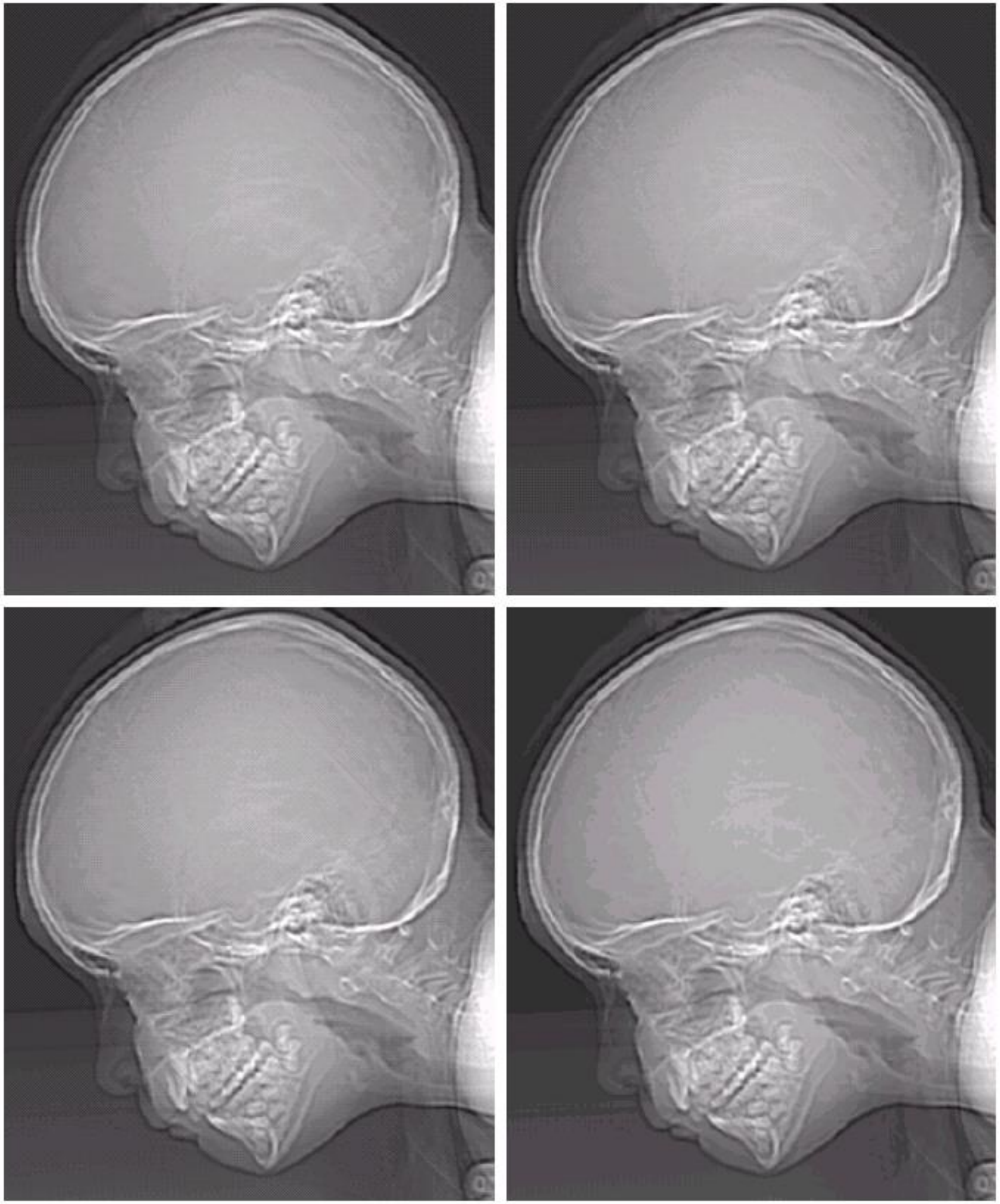
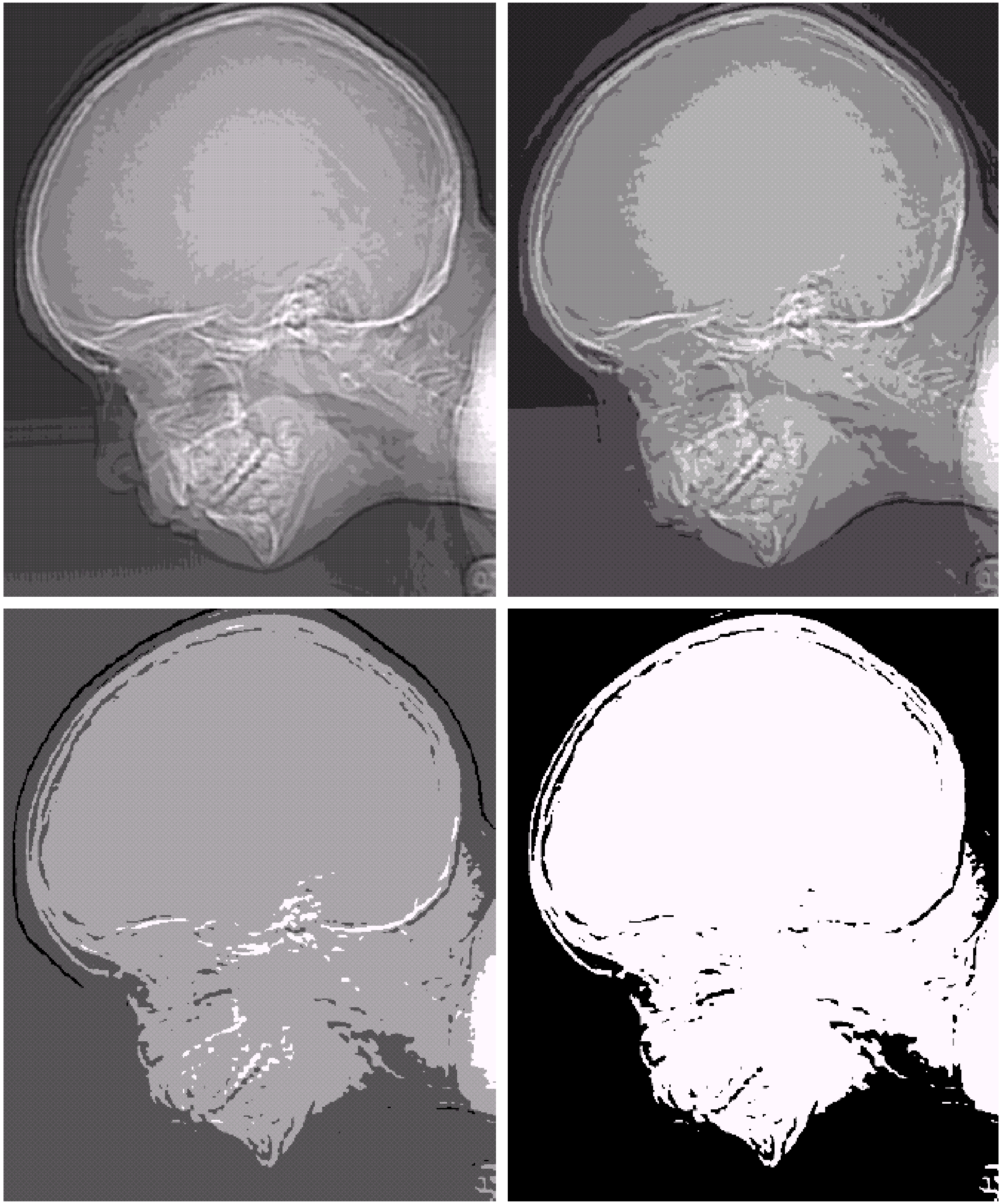
- **Spatial and Gray-Level Resolution**
  - Figure 2.19 shows an image of size 1024×1024 pixels whose gray levels are represented by 8 bits.
  - The simplest way to compare these effects is to bring all the subsampled images up to size 1024×1024 by row and column pixel replication.
    - Figure. 2.20

FIGURE 2.20 Typical effects of reducing spatial resolution. Images shown at: (a) 1250 dpi, (b) 300 dpi, (c) 150 dpi, and (d) 72 dpi. The thin black borders were added for clarity. They are not part of the data.



a	b
c	d

FIGURE 2.21 (a) 452 × 374, 256-level image. (b)–(d) Image displayed in 128, 64, and 32 gray levels, while keeping the spatial resolution constant.



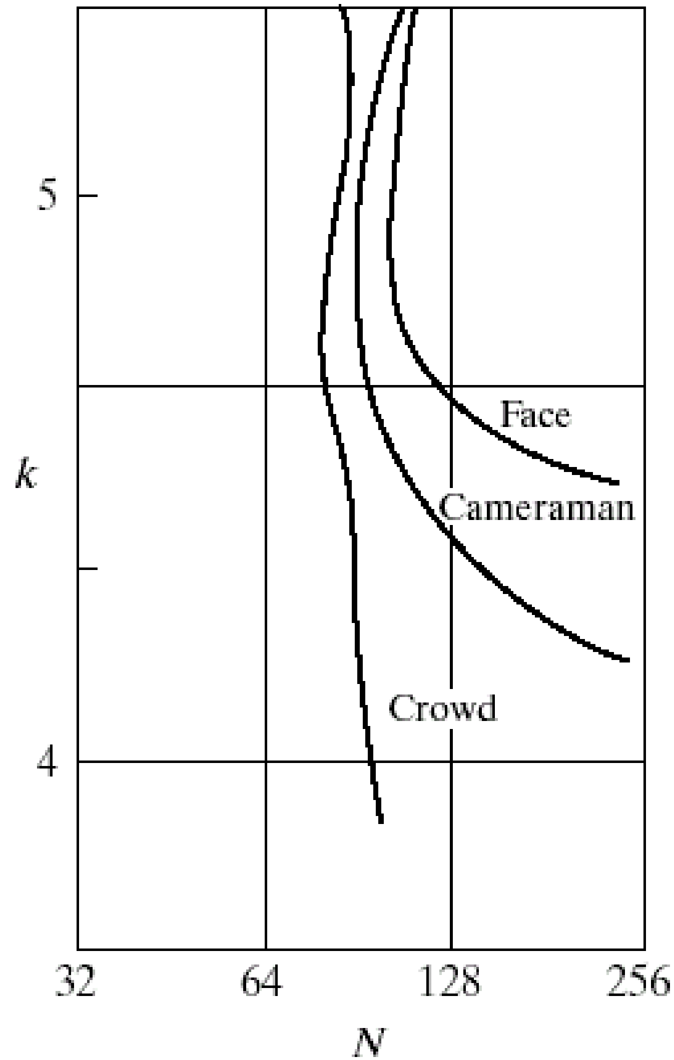
e f
g h

FIGURE 2.21 *(Continued)* (e)–(h) Image displayed in 16, 8, 4, and 2 gray levels. (Original courtesy of Dr. David R. Pickens, Department of Radiology & Radiological Sciences, Vanderbilt University Medical Center.)

a b c

FIGURE 2.22 (a) Image with a low level of detail. (b) Image with a medium level of detail. (c) Image with a relatively large amount of detail. (Image (b) courtesy of the Massachusetts Institute of Technology.)

FIGURE 2.23 Representative isopreference curves for the three types of images in Fig. 2.22.



- **EXAMPLE 2.3:**
  - Typical effects of varying the number of gray levels in a digital image.
    - FIGURE 2.21
- **Zooming and Shrinking Digital Images(FIGURE 2.25)**
  - *Nearest neighbor interpolation*
  - *Pixel replication*
  - *Bilinear interpolation*

$$v(x', y') = ax' + by' + cx'y' + d \tag{2.4-6}$$

let $v(x', y')$ denote the gray level assigned

a b c
d e f

FIGURE 2.24 (a) Image reduced to 72 dpi and zoomed back to its original size (3692 × 2812 pixels) using nearest neighbor interpolation. This figure is the same as Fig. 2.20(d). (b) Image shrunk and zoomed using bilinear interpolation. (c) Same as (b) but using bicubic interpolation. (d)–(f) Same sequence, but shrinking down to 150 dpi instead of 72 dpi [Fig. 2.24(d) is the same as Fig. 2.20(c)]. Compare Figs. 2.24(e) and (f), especially the latter, with the original image in Fig. 2.20(a).

2.5.1 Neighbors of a Pixel

A pixel p at coordinates (x, y) has four *horizontal* and *vertical* neighbors whose coordinates are given by

$$(x+1, y), (x-1, y), (x, y+1), (x, y-1)$$

This set of pixels, called the *4-neighbors* of p, is denoted by $N_4(p)$. Each pixel is a unit distance from (x, y), and some of the neighbors of p lie outside the digital image if (x, y) is on the border of the image.

The four *diagonal* neighbors of p have coordinates

$$(x+1, y+1), (x+1, y-1), (x-1, y+1), (x-1, y-1)$$

and are denoted by $N_D(p)$. These points, together with the 4-neighbors, are called the *8-neighbors* of p, denoted by $N_8(p)$. As before, some of the points in $N_D(p)$ and $N_8(p)$ fall outside the image if (x, y) is on the border of the image.

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- **Adjacency**
  - three types of adjacency:

(a) 4-*adjacency*. Two pixels p and q with values from V are 4-adjacent if q is in the set $N_4(p)$.

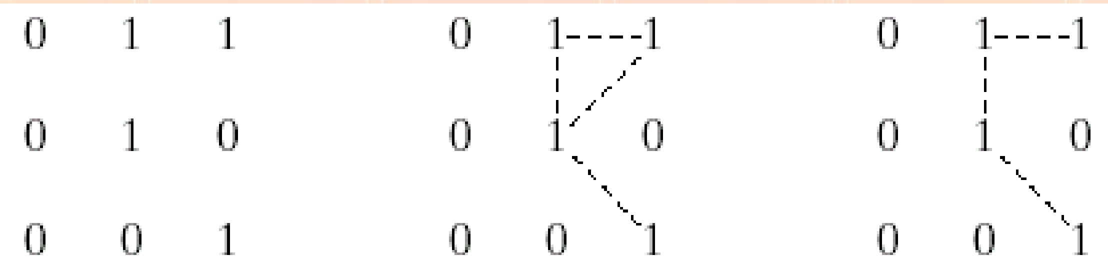
(b) 8-*adjacency*. Two pixels p and q with values from V are 8-adjacent if q is in the set $N_8(p)$.

(c) *m-adjacency* (mixed adjacency). Two pixels p and q with values from V are m-adjacent if

(i) q is in $N_4(p)$, *or*

(ii) q is in $N_D(p)$ *and* the set $N_4(p) \cap N_4(q)$ has no pixels whose values are from V.

  - Figs. 2-26



a b c

FIGURE 2.26 (a) Arrangement of pixels; (b) pixels that are 8-adjacent (shown dashed) to the center pixel; (c) *m*-adjacency.

Basic Relationships Between Pixels

- **Connectivity**
  - Two pixels p and q are said to be *connected* in *S* if there exists a path between them consisting entirely of pixels in *S*.
  - For any pixel p in *S*, the *set* of pixels that are connected to it in *S* is called a *connected component* of *S*.
  - If it only has one connected component, then set *S* is called a *connected set*.
- **Regions**
  - Let *R* be a subset of pixels in an image.We call *R* a *region* of the image if *R* is a connected set.
- **Boundaries**
  - The *boundary* (also called *border* or *contour*) of a region *R* is the set of pixels in the region that have one or more neighbors that are not in *R*.

- **Distance Measures**

For pixels p, q, and z, with coordinates (x, y), (s, t), and (v, w), respectively, D is a *distance function* or *metric* if

(a) $D(p, q) \geq 0 \quad (D(p, q) = 0 \quad \text{iff} \quad p = q)$,
(b) $D(p, q) = D(q, p)$, and
(c) $D(p, z) \leq D(p, q) + D(q, z)$.

The *Euclidean distance* between p and q is defined as

$$D_e(p, q) = \left[(x - s)^2 + (y - t)^2\right]^{\frac{1}{2}}. \tag{2.5-1}$$

For this distance measure, the pixels having a distance less than or equal to some value r from (x, y) are the points contained in a disk of radius r centered at (x, y).

The D_4 *distance* (also called *city-block distance*) between p and q is defined as

$$D_4(p, q) = |x - s| + |y - t|. \tag{2.5-2}$$

2.5 Some Basic Relationships Between Pixels

In this case, the pixels having a D_4 distance from (x, y) less than or equal to some value r form a diamond centered at (x, y). For example, the pixels with D_4 distance ≤ 2 from (x, y) (the center point) form the following contours of constant distance:

$$\begin{matrix} & & 2 & & \\ & 2 & 1 & 2 & \\ 2 & 1 & 0 & 1 & 2 \\ & 2 & 1 & 2 & \\ & & 2 & & \end{matrix}$$

The pixels with $D_4 = 1$ are the 4-neighbors of (x, y).

2.5 Some Basic Relationships Between Pixels

The D_8 *distance* (also called *chessboard distance*) between p and q is defined as

$$D_8(p, q) = \max(|x - s|, |y - t|). \quad (2.5\text{-}3)$$

In this case, the pixels with D_8 distance from (x, y) less than or equal to some value r form a square centered at (x, y). For example, the pixels with D_8 distance ≤ 2 from (x, y) (the center point) form the following contours of constant distance:

2	2	2	2	2
2	1	1	1	2
2	1	0	1	2
2	1	1	1	2
2	2	2	2	2

The pixels with $D_8 = 1$ are the 8-neighbors of (x, y).

2.6 Mathematical Tools

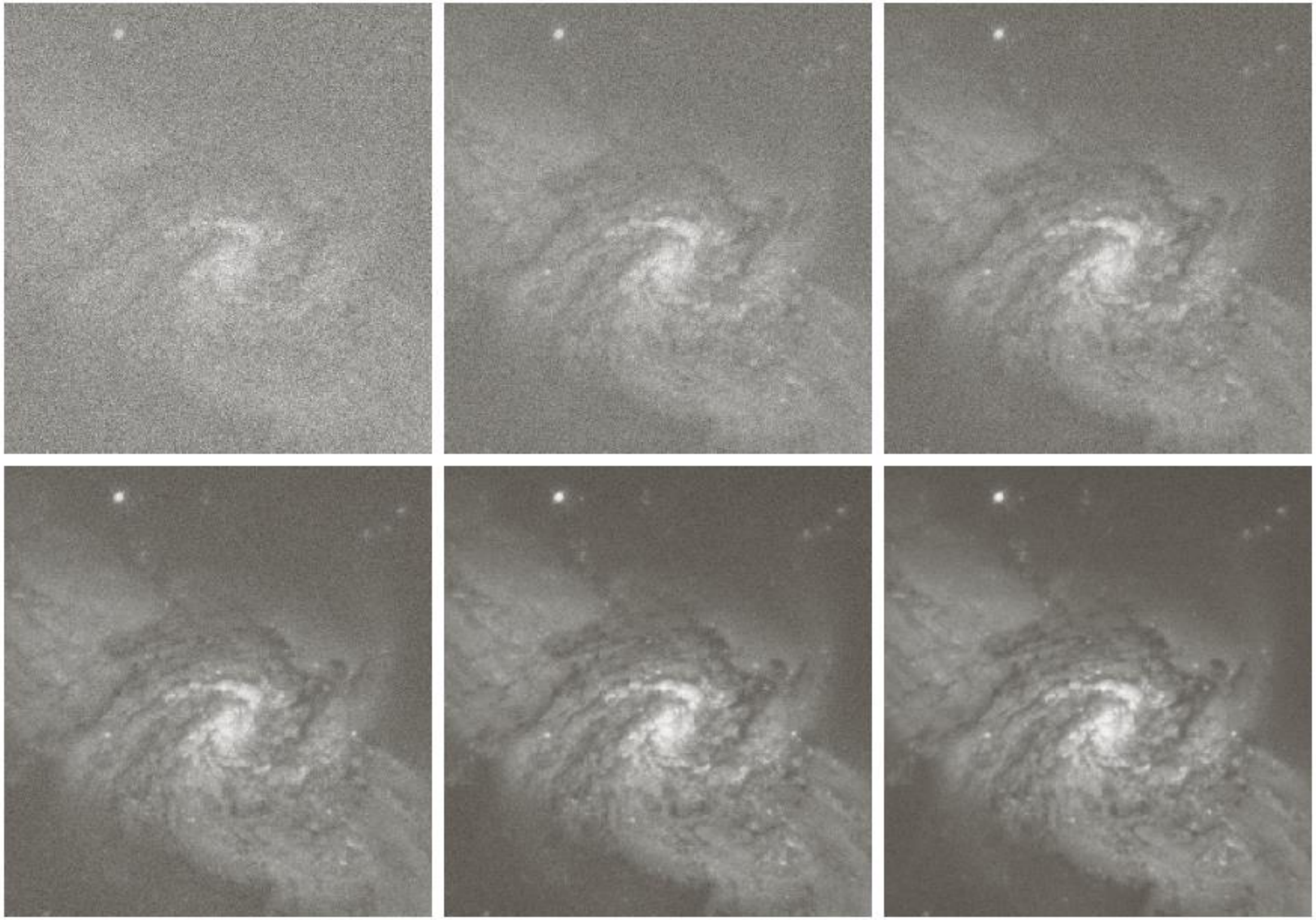
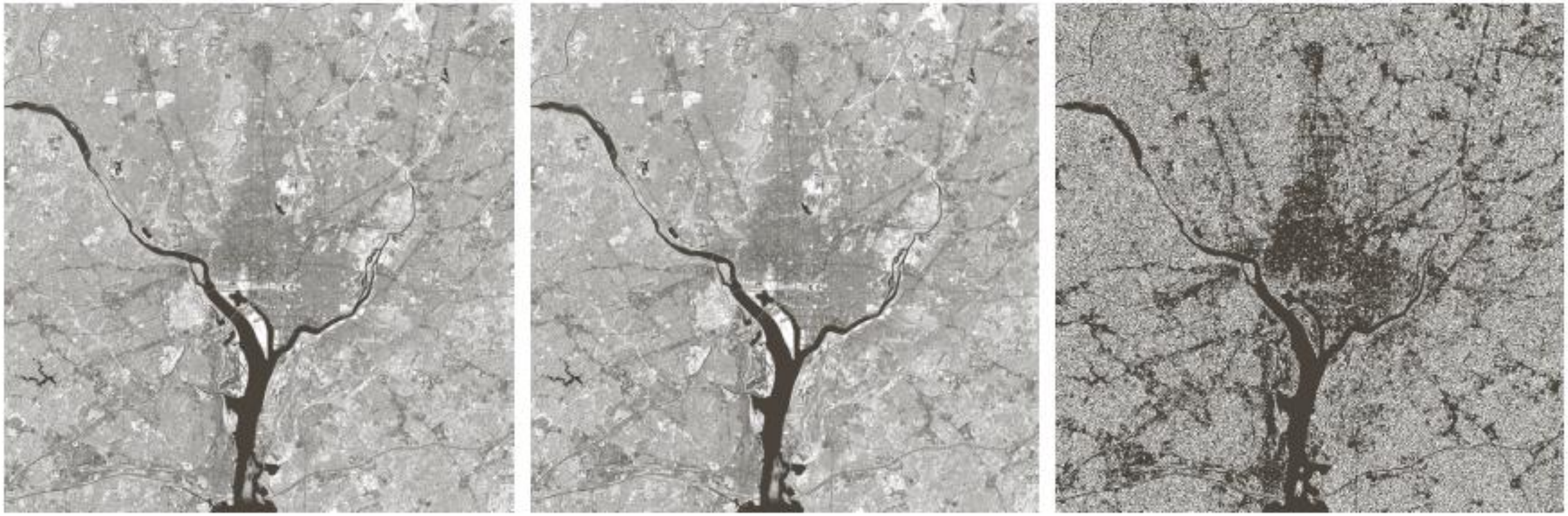


FIGURE 2.26 (a) Image of Galaxy Pair NGC 3314 corrupted by additive Gaussian noise. (b)–(f) Results of averaging 5, 10, 20, 50, and 100 noisy images, respectively. (Original image courtesy of NASA.)

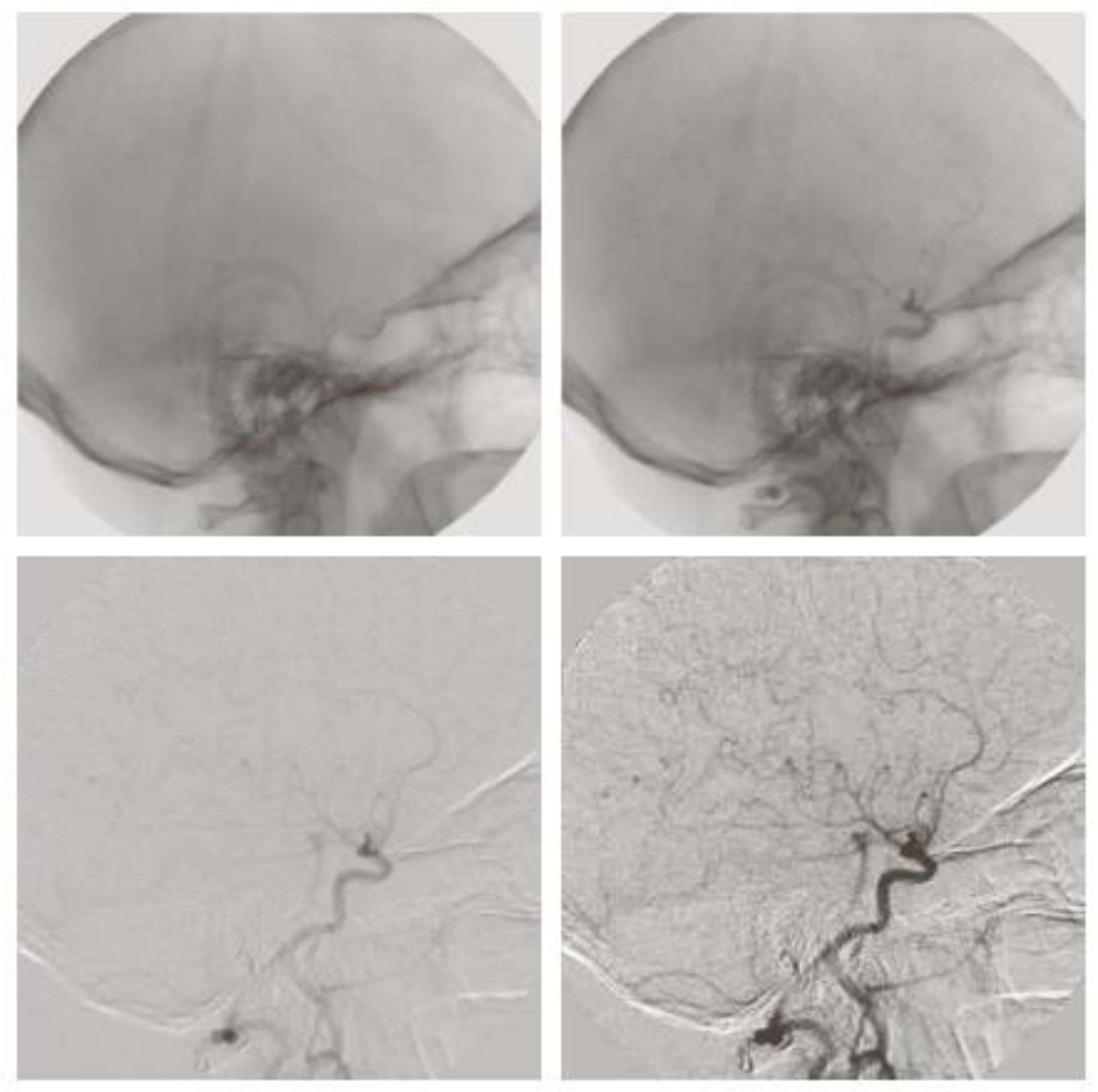
2.6 Mathematical Tools



a b c

FIGURE 2.27 (a) Infrared image of the Washington, D.C. area. (b) Image obtained by setting to zero the least significant bit of every pixel in (a). (c) Difference of the two images, scaled to the range [0, 255] for clarity.

2.6 Mathematical Tools



a b
c d

FIGURE 2.28 Digital subtraction angiography. (a) Mask image. (b) A live image. (c) Difference between (a) and (b). (d) Enhanced difference image. (Figures (a) and (b) courtesy of The Image Sciences Institute, University Medical Center, Utrecht, The Netherlands.)

2.6 Mathematical Tools

a b c

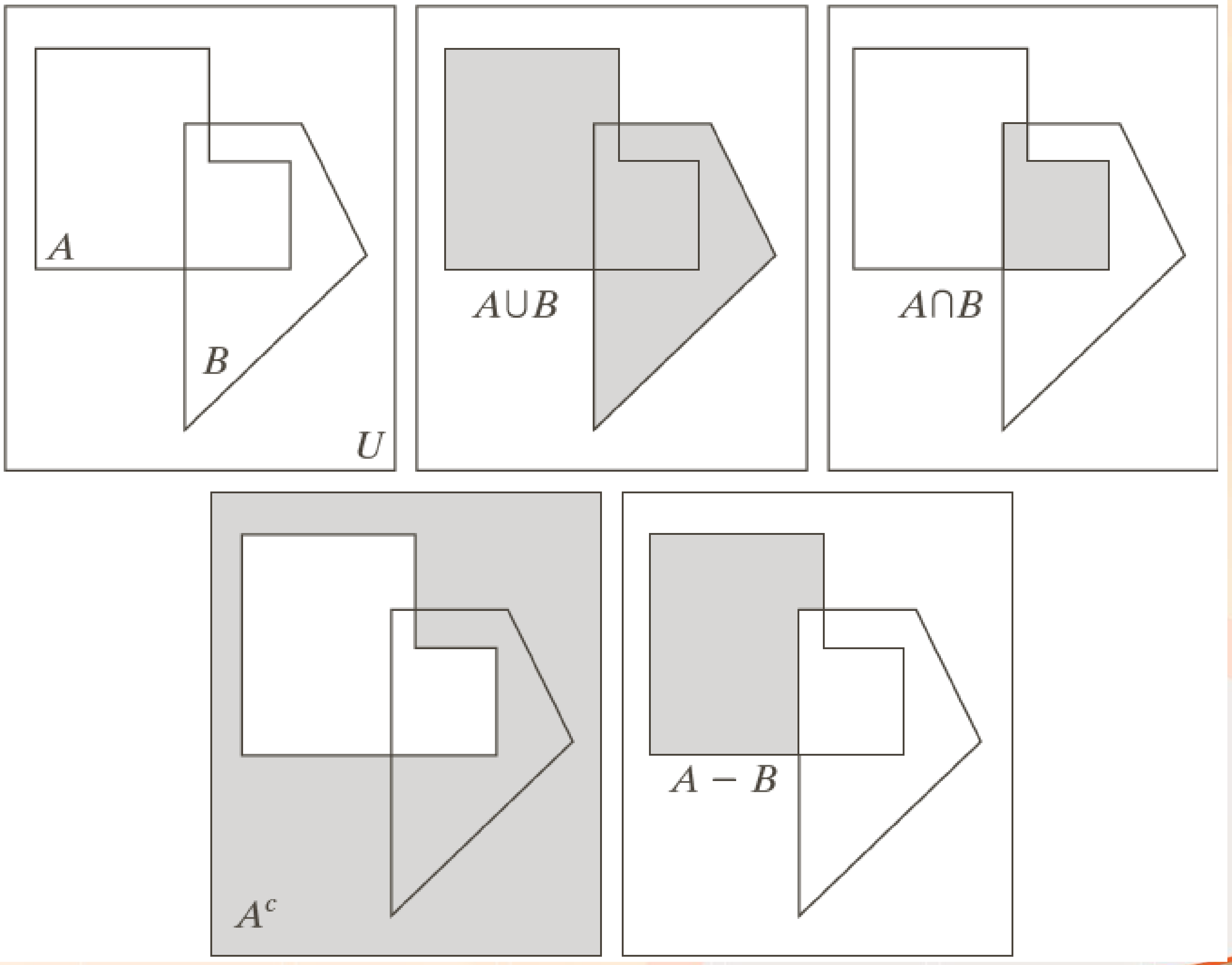
FIGURE 2.29 Shading correction. (a) Shaded SEM image of a tungsten filament and support, magnified approximately 130 times. (b) The shading pattern. (c) Product of (a) by the reciprocal of (b). (Original image courtesy of Mr. Michael Shaffer, Department of Geological Sciences, University of Oregon, Eugene.)

2.6 Mathematical Tools

a b c

FIGURE 2.30 (a) Digital dental X-ray image. (b) ROI mask for isolating teeth with fillings (white corresponds to 1 and black corresponds to 0). (c) Product of (a) and (b).

2.6 Mathematical Tools



a b c
d e

FIGURE 2.31
(a) Two sets of coordinates, A and B, in 2-D space. (b) The union of A and B. (c) The intersection of A and B. (d) The complement of A. (e) The difference between A and B. In (b)–(e) the shaded areas represent the member of the set operation indicated.

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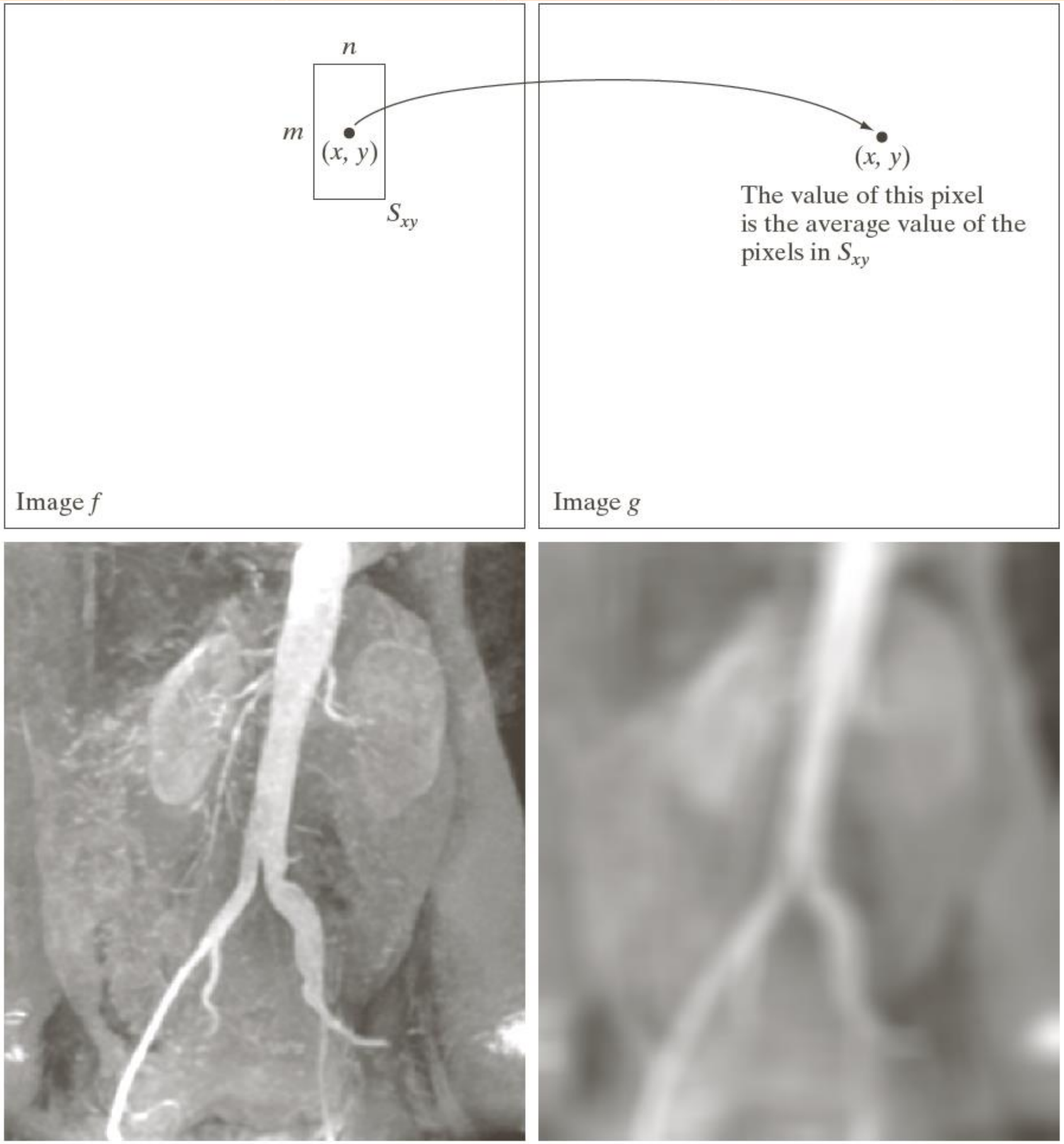

a	b
c	d

FIGURE 2.35 Local averaging using neighborhood processing. The procedure is illustrated in (a) and (b) for a rectangular neighborhood. (c) The aortic angiogram discussed in Section 1.3.2. (d) The result of using Eq. (2.6-21) with $m = n = 41$. The images are of size 790×686 pixels.

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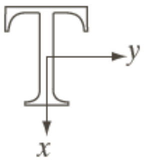
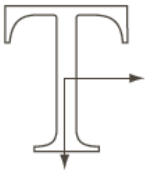
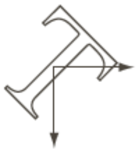
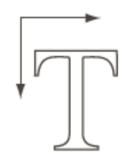
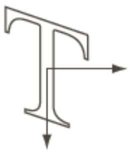
TABLE 2.2
Affine transformations based on Eq. (2.6.–23).

Transformation Name	Affine Matrix, T	Coordinate Equations	Example
Identity	$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	$x = v$ $y = w$	
Scaling	$\begin{bmatrix} c_x & 0 & 0 \\ 0 & c_y & 0 \\ 0 & 0 & 1 \end{bmatrix}$	$x = c_x v$ $y = c_y w$	
Rotation	$\begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$	$x = v\cos\theta - w\sin\theta$ $y = v\cos\theta + w\sin\theta$	
Translation	$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ t_x & t_y & 1 \end{bmatrix}$	$x = v + t_x$ $y = w + t_y$	
Shear (vertical)	$\begin{bmatrix} 1 & 0 & 0 \\ s_v & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	$x = v + s_v w$ $y = w$	
Shear (horizontal)	$\begin{bmatrix} 1 & s_h & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	$x = v$ $y = s_h v + w$	

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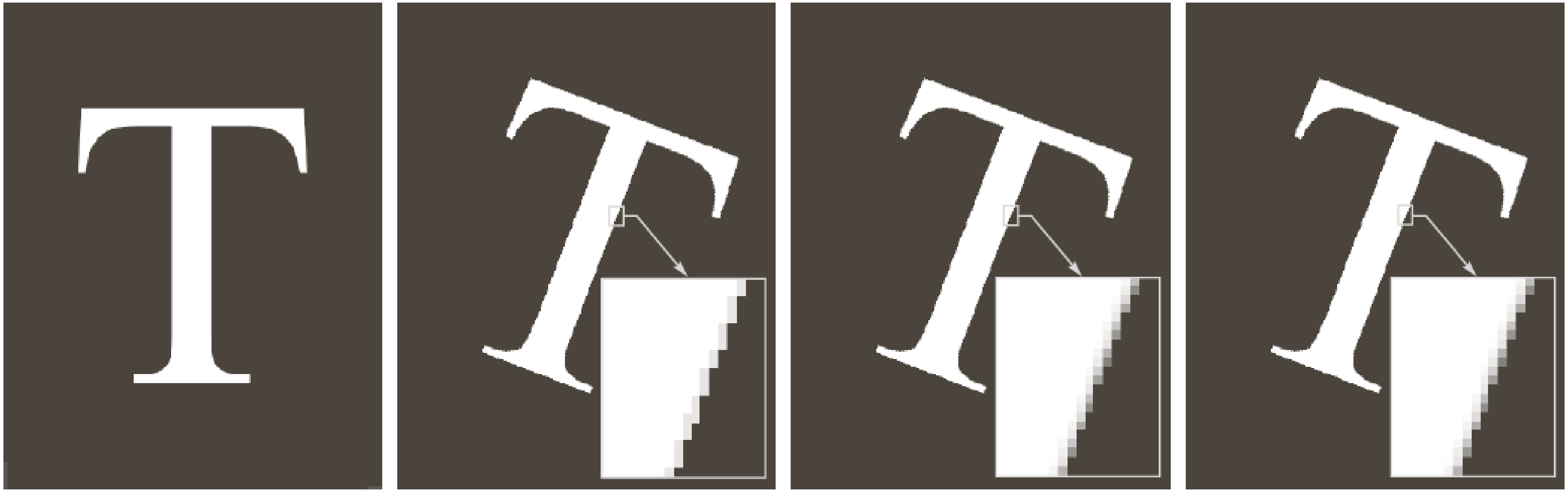

a b c d

FIGURE 2.36 (a) A 300 dpi image of the letter T. (b) Image rotated 21° clockwise using nearest neighbor interpolation to assign intensity values to the spatially transformed pixels. (c) Image rotated 21° using bilinear interpolation. (d) Image rotated 21° using bicubic interpolation. The enlarged sections show edge detail for the three interpolation approaches.

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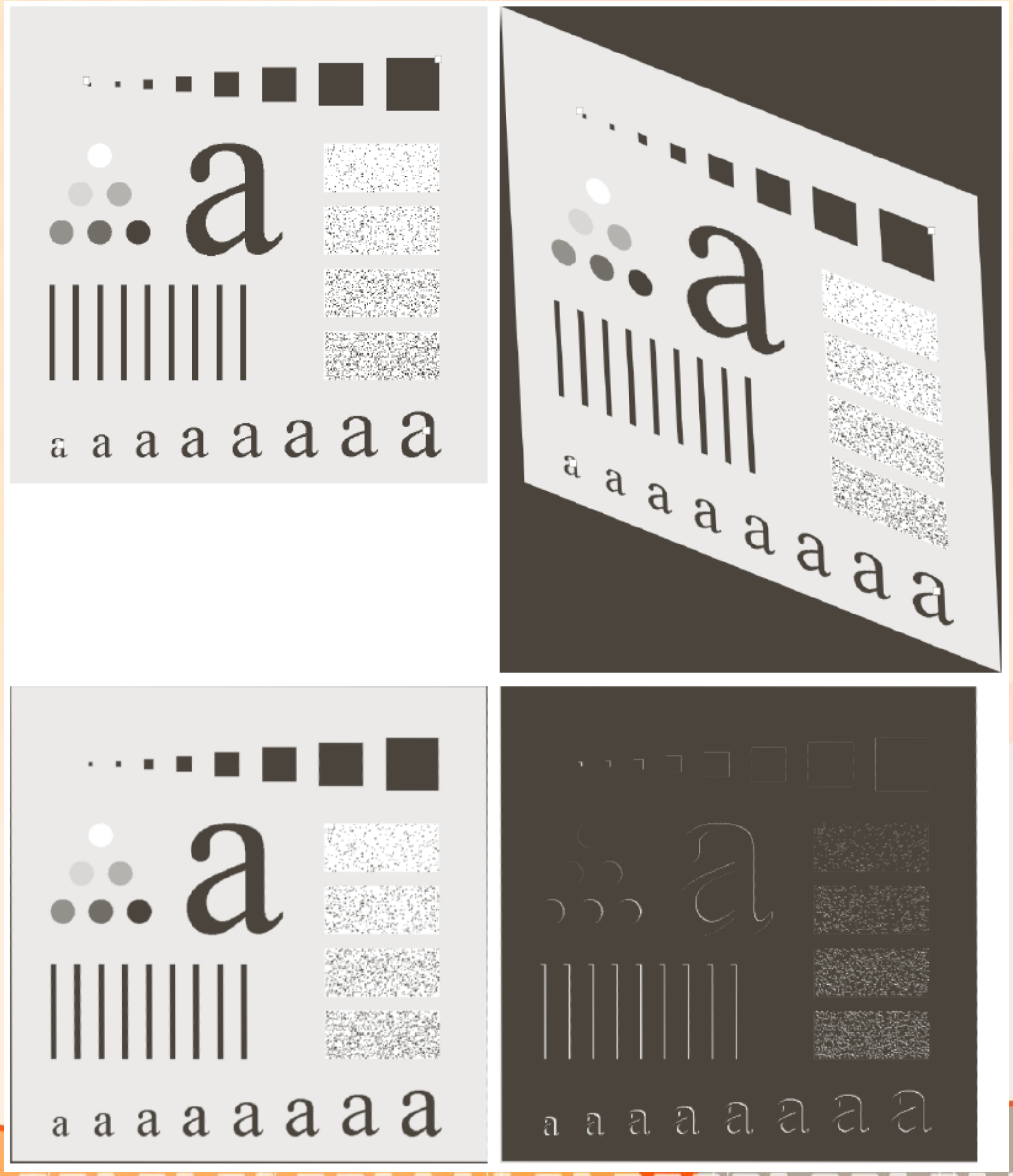

a	b
c	d

FIGURE 2.37 Image registration. (a) Reference image. (b) Input (geometrically distorted image). Corresponding tie points are shown as small white squares near the corners. (c) Registered image (note the errors in the borders). (d) Difference between (a) and (c), showing more registration errors.

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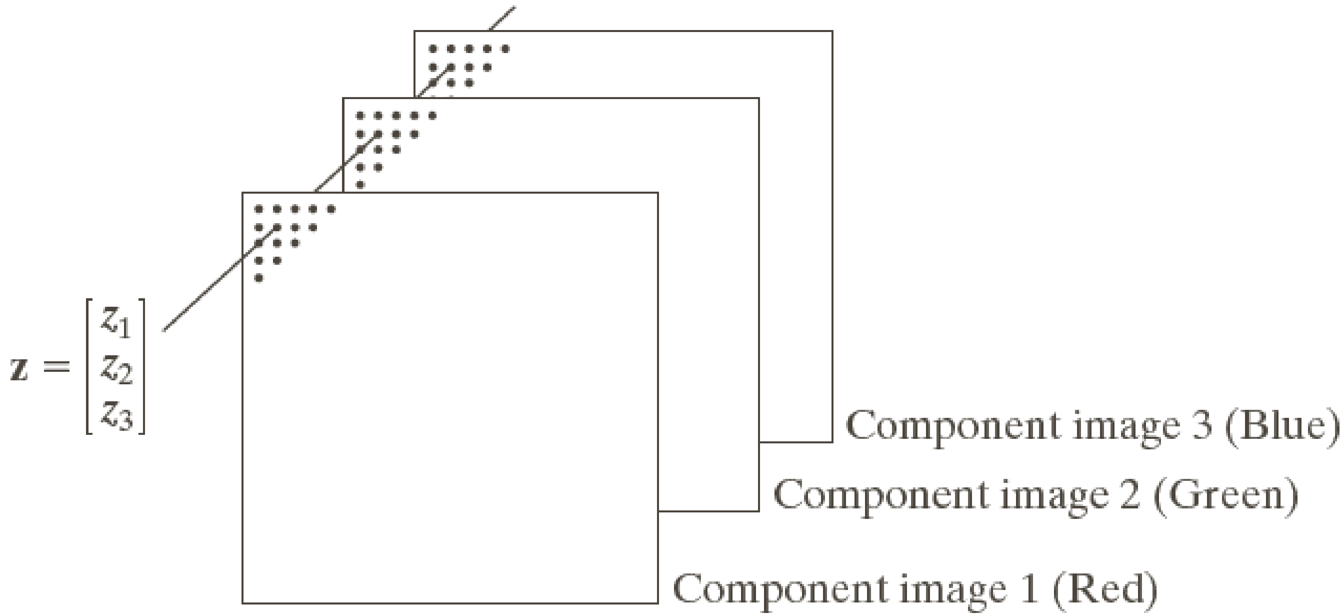

FIGURE 2.38 Formation of a vector from corresponding pixel values in three RGB component images.

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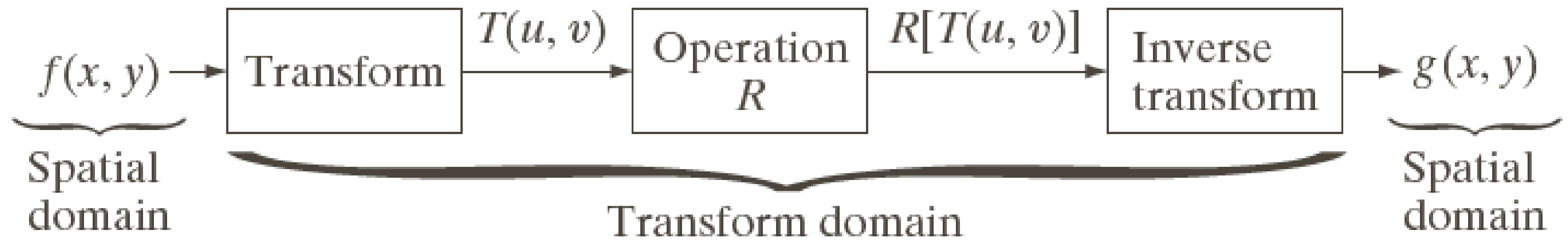

FIGURE 2.39 General approach for operating in the linear transform domain.

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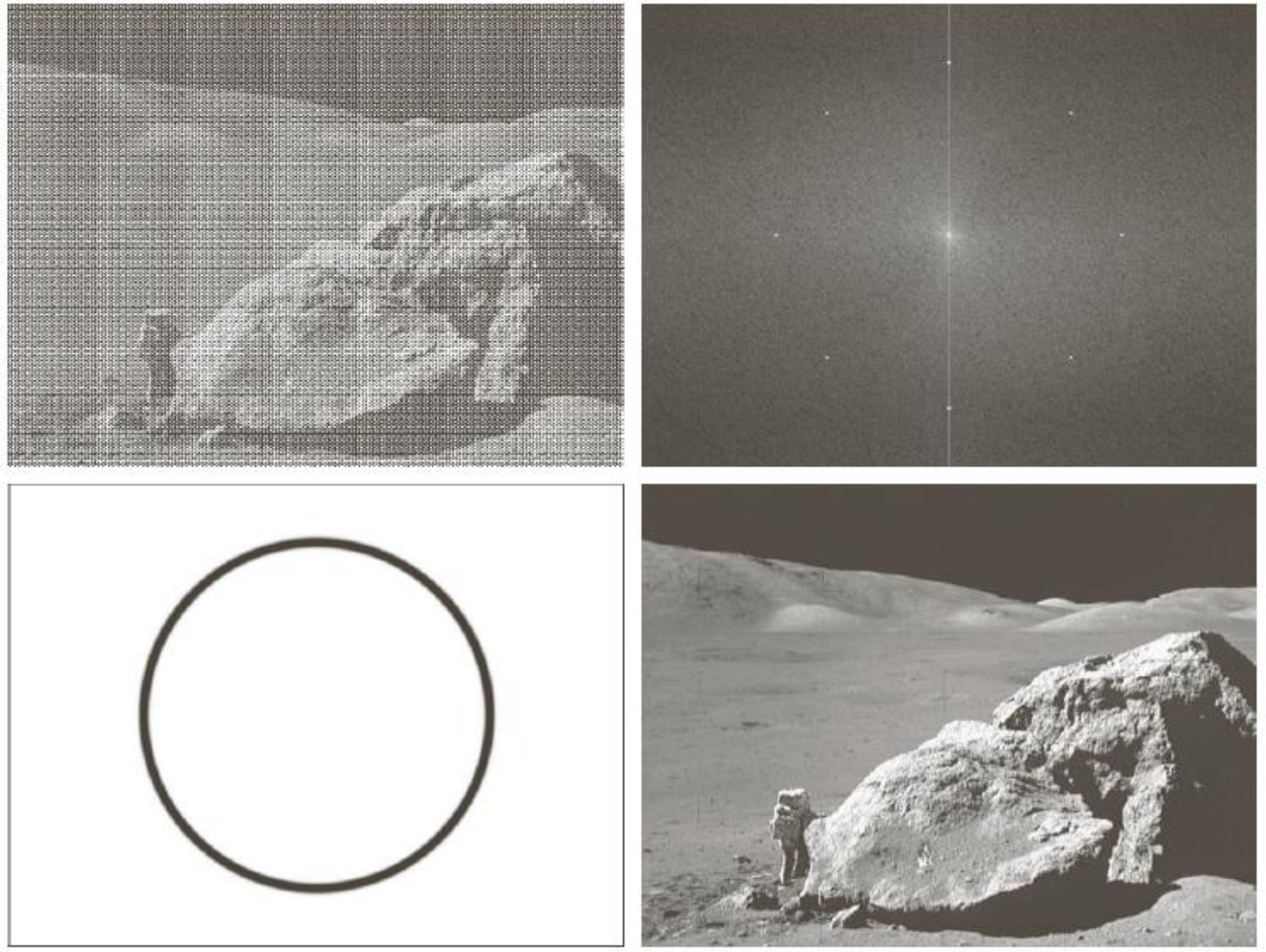

a b
c d

FIGURE 2.40 (a) Image corrupted by sinusoidal interference. (b) Magnitude of the Fourier transform showing the bursts of energy responsible for the interference. (c) Mask used to eliminate the energy bursts. (d) Result of computing the inverse of the modified Fourier transform. (Original image courtesy of NASA.)

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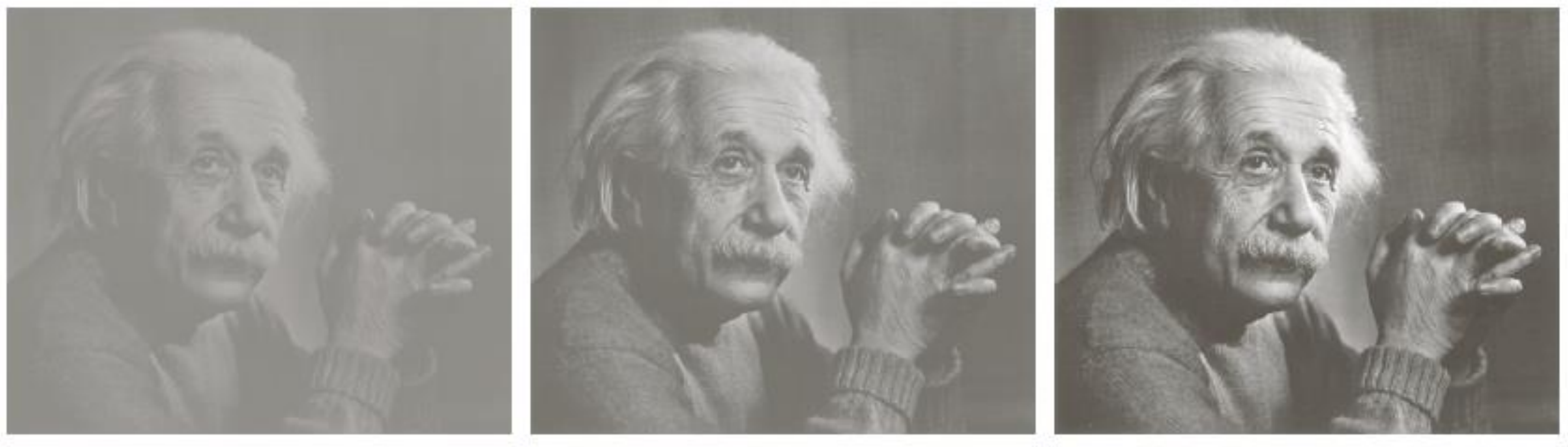

a b c

FIGURE 2.41 Images exhibiting (a) low contrast, (b) medium contrast, and (c) high contrast.