Image
Video
Processing
Laboratory

Chapter 7

Wavelets and Multiresolution Processing

Background

- Fundamental motivation for multiresolution processing
  - Fig. 7.1
- Image pyramid – representation of images at more than one resolution
  - Fig. 7.2(a)
  - *Prediction residual pyramids* – Fig. 7.2(b)

物體小或對比低→高解析度
物體大或對比高→低解析度

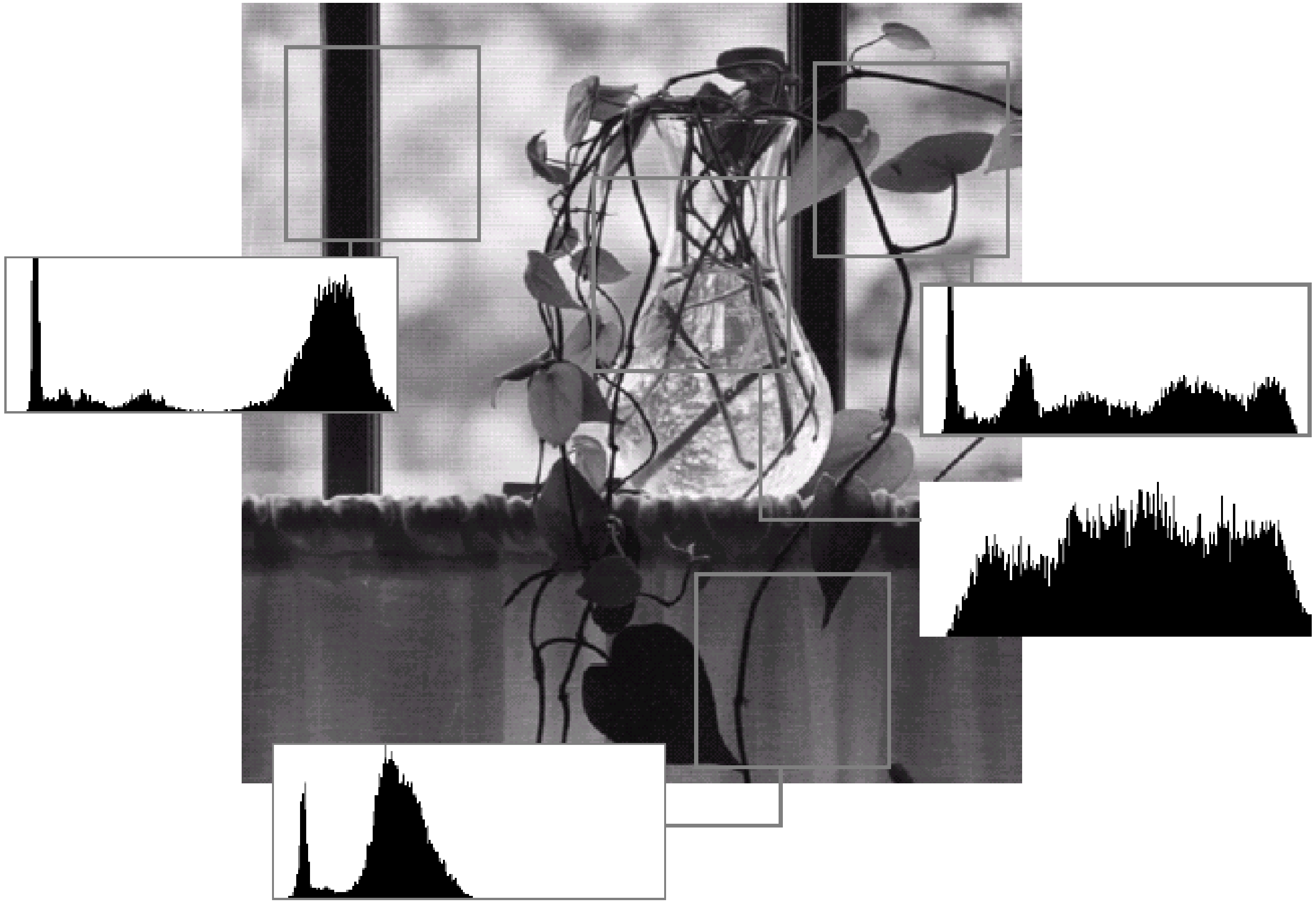
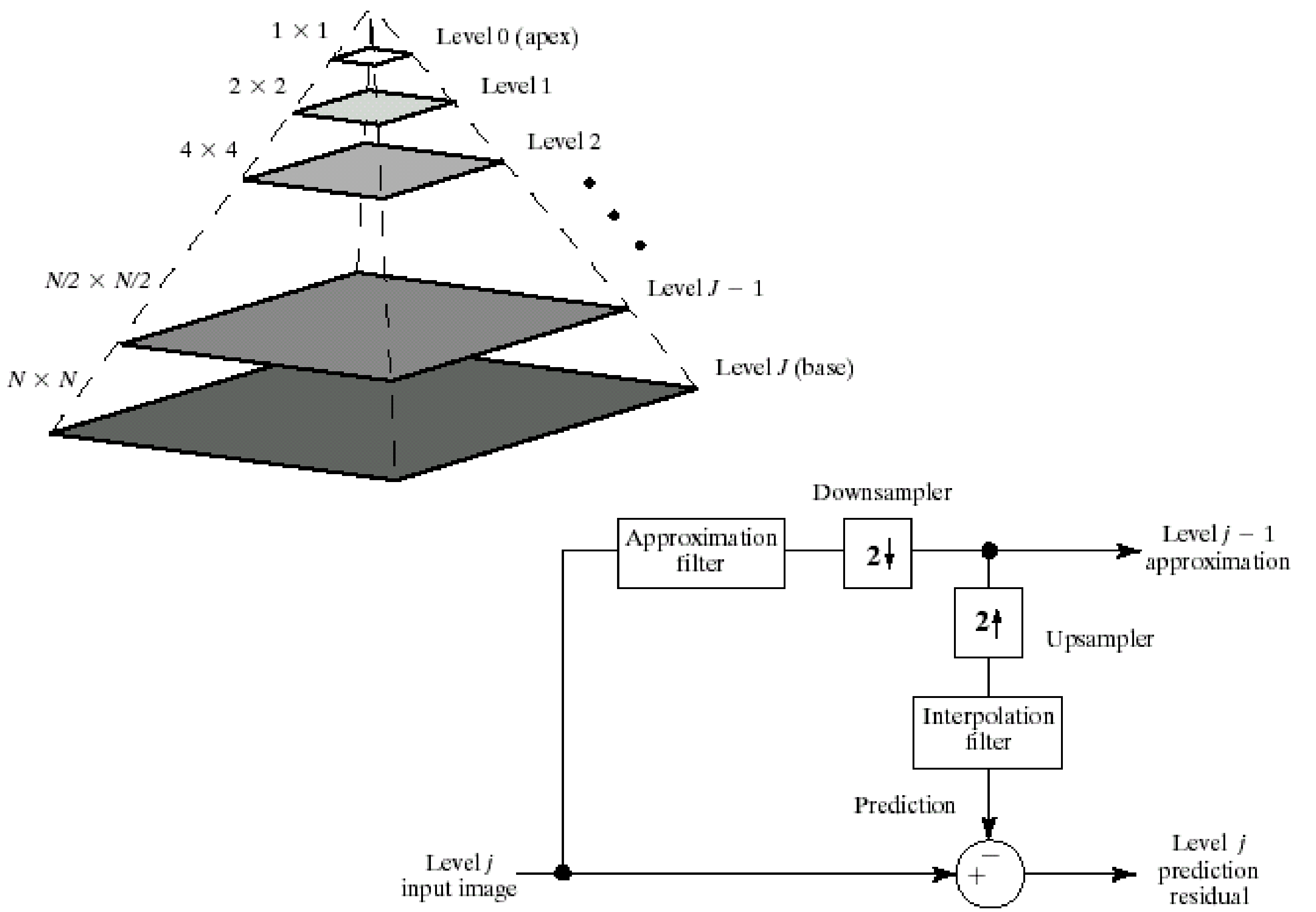


FIGURE 7.1 A natural image and its local histogram variations.



a b

FIGURE 7.2 (a) A pyramidal image structure and (b) system block diagram for creating it.

- Example 7.1-Gaussian and Laplacian pyramids.

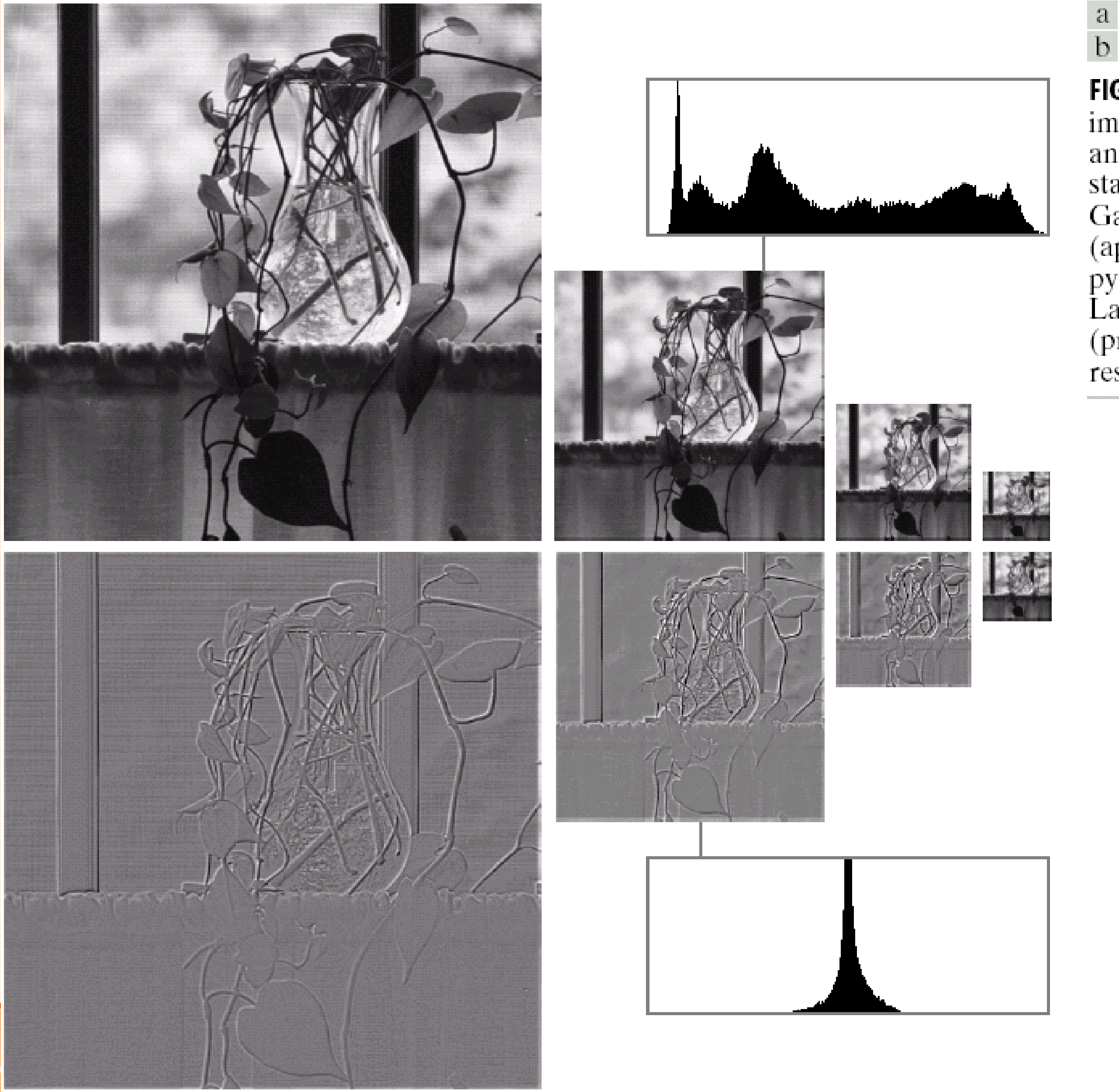
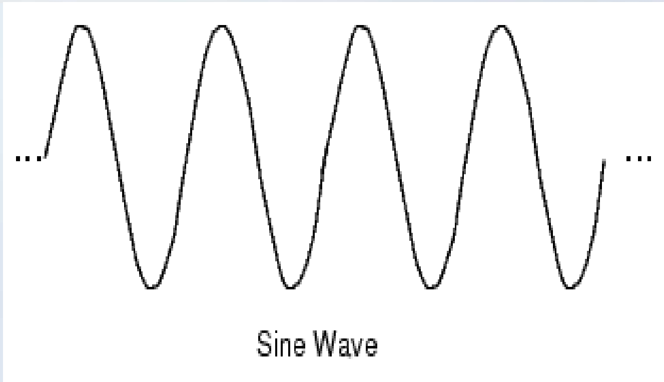


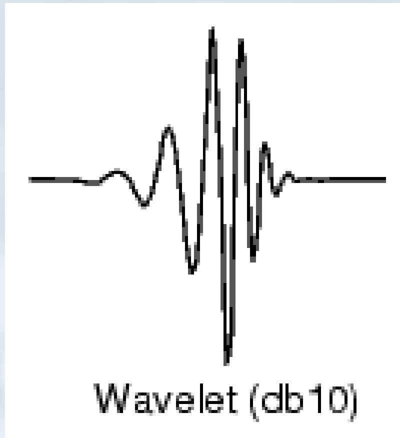
FIGURE 7.3 Two image pyramids and their statistics: (a) a Gaussian (approximation) pyramid and (b) a Laplacian (prediction residual) pyramid.

Preview

- Fourier transform
  - Basis functions are sinusoids



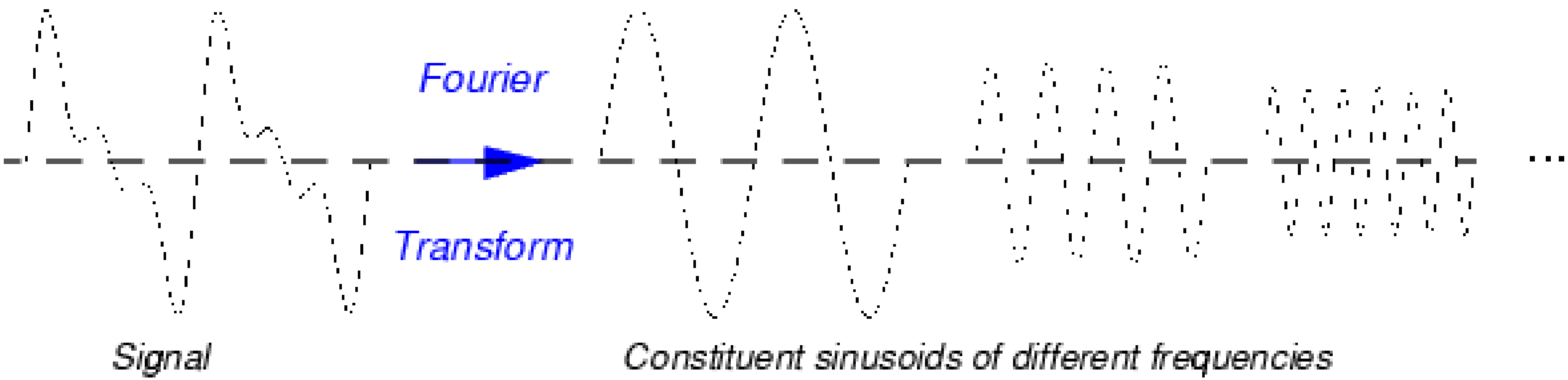
- Wavelet transform 小波
  - Basis functions are small waves, of varying frequency and limited duration



Signal representation (1)

- Fourier transform

$$f(x) = \int_{-\infty}^{\infty} F(u) e^{j2\pi ux} du$$

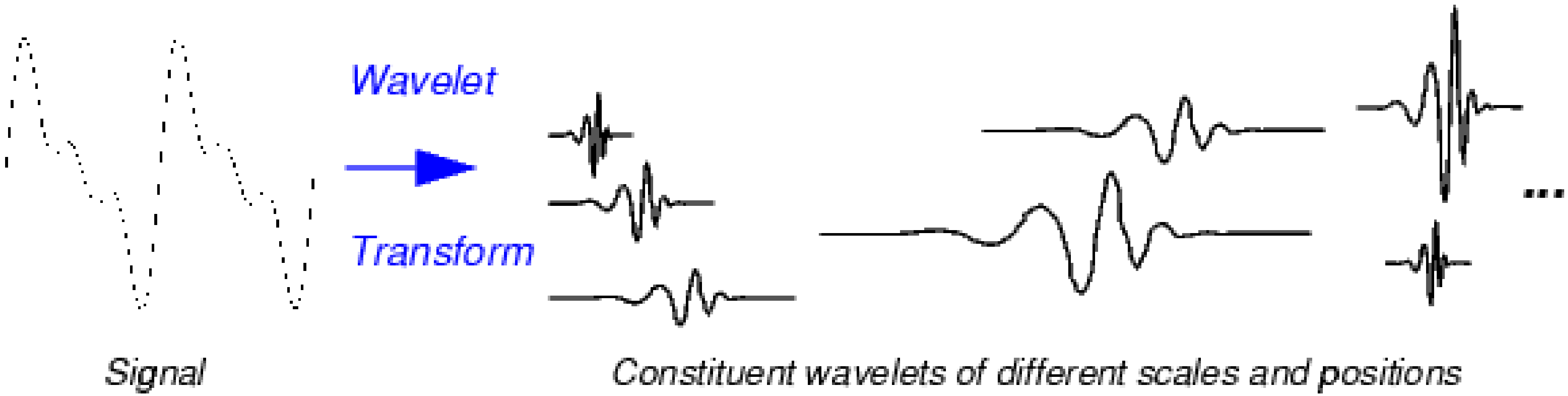


Sinusoid has unlimited duration

Signal representation (2)

- Wavelet transform

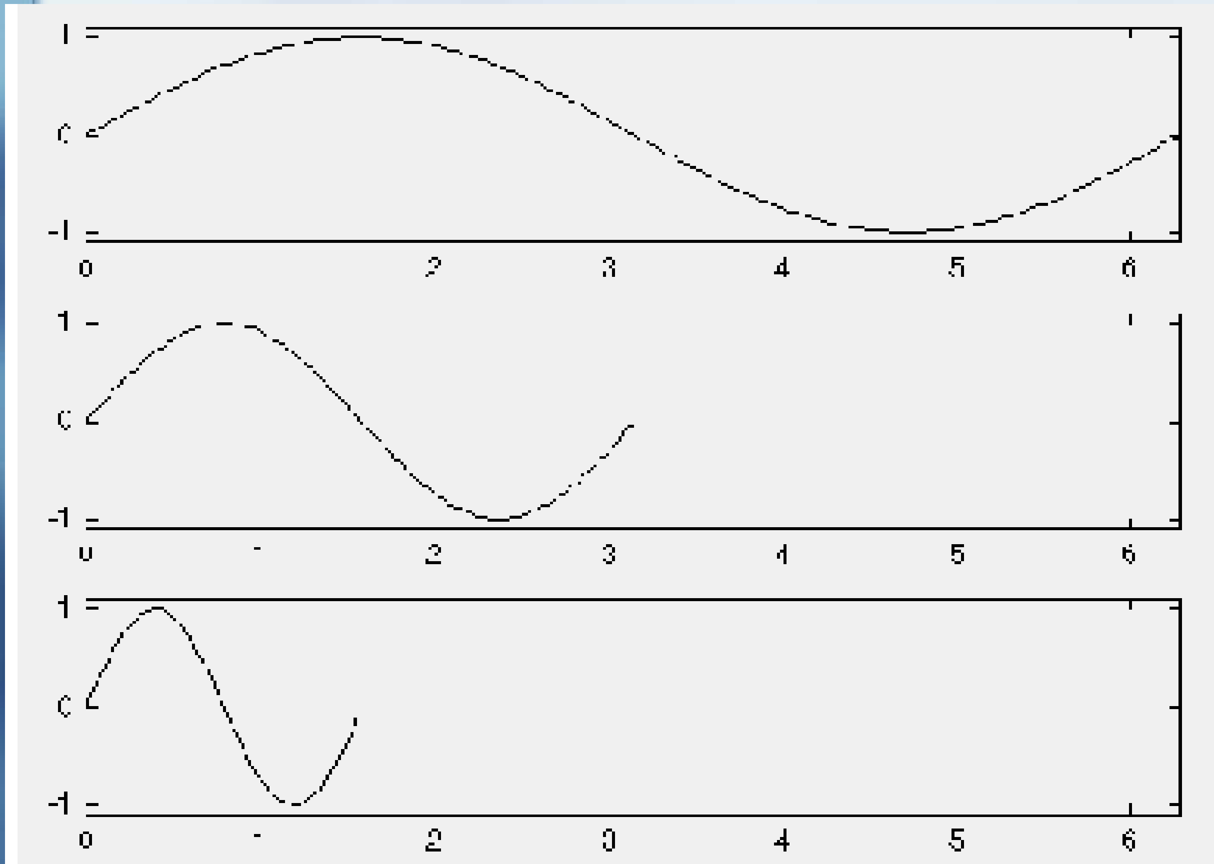
$$f(t) = \int_{-\infty}^{\infty} C(scale, position)\psi(scale, position, t)dt$$



A wavelet has compact support (limited duration)

Scaling (1)

- What is the scale factor?



$f(t) = \sin(t); \quad a = 1$

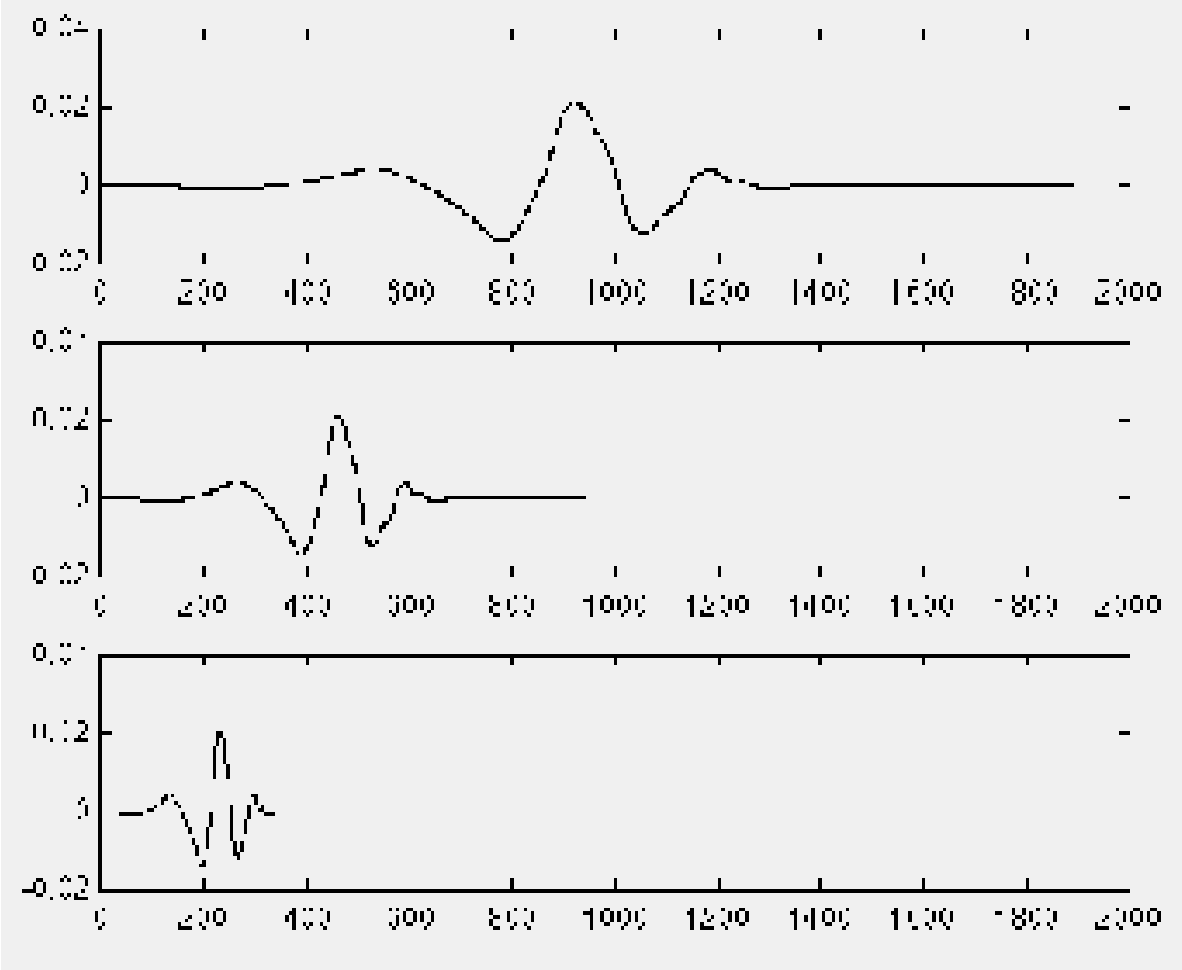
$f(t) = \sin(2t); \quad a = \frac{1}{2}$

$f(t) = \sin(4t); \quad a = \frac{1}{4}$

Ex#1: Plot the above diagrams (hint: plot command)

Scaling (2)

- Scaling for wavelet function



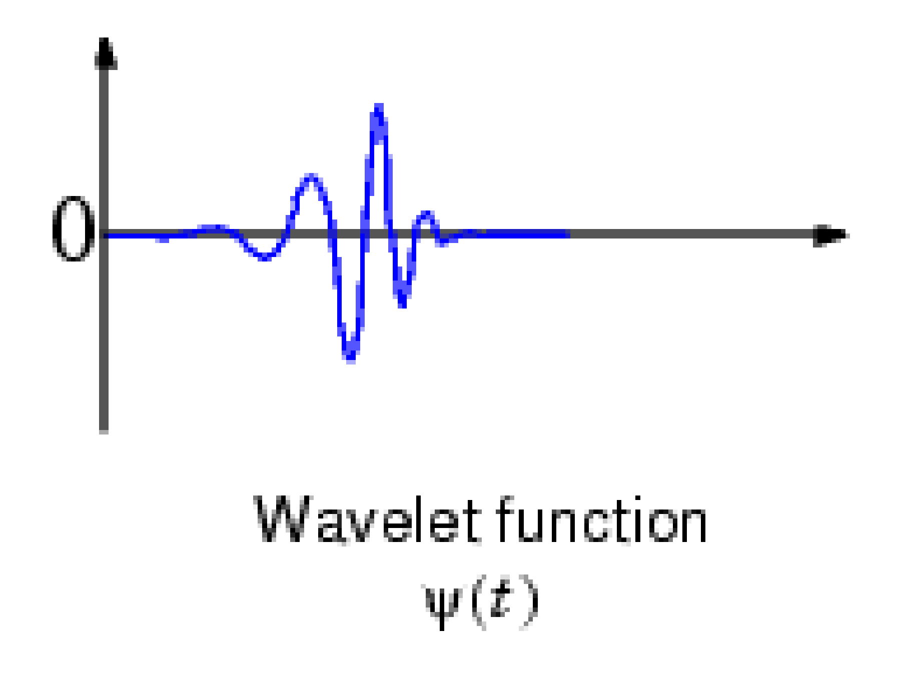
$$f(t) = \psi(t) \quad ; \quad a = 1$$

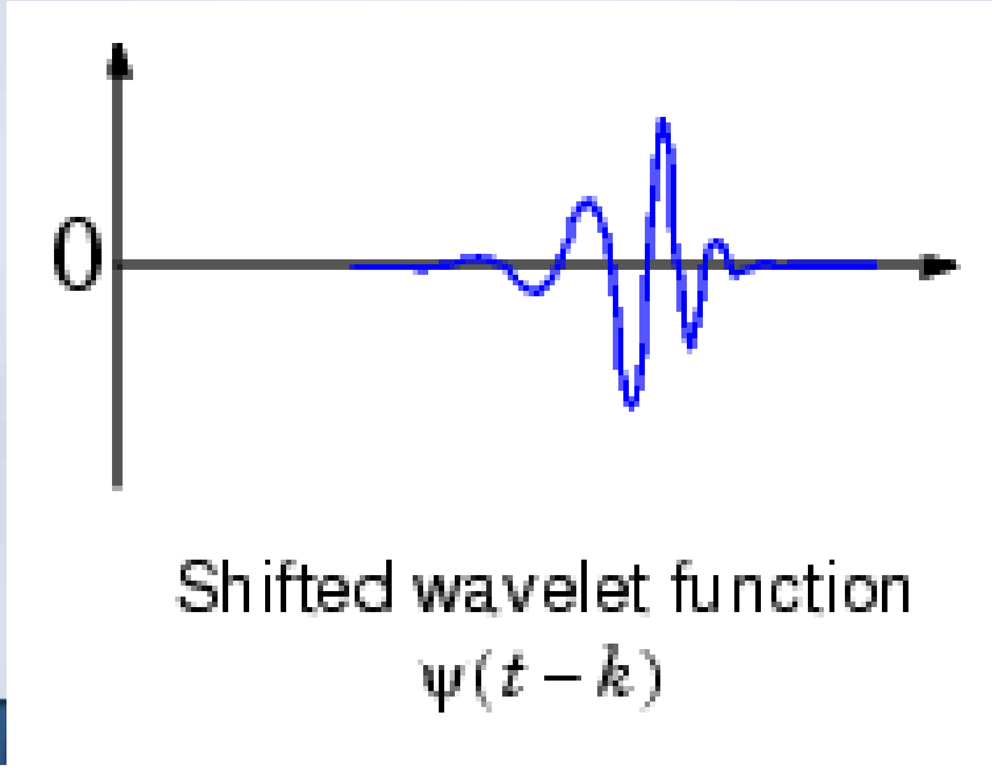
$$f(t) = \psi(2t) \; ; \quad a = \frac{1}{2}$$

$$f(t) = \psi(4t) \; ; \quad a = \frac{1}{4}$$

Shift

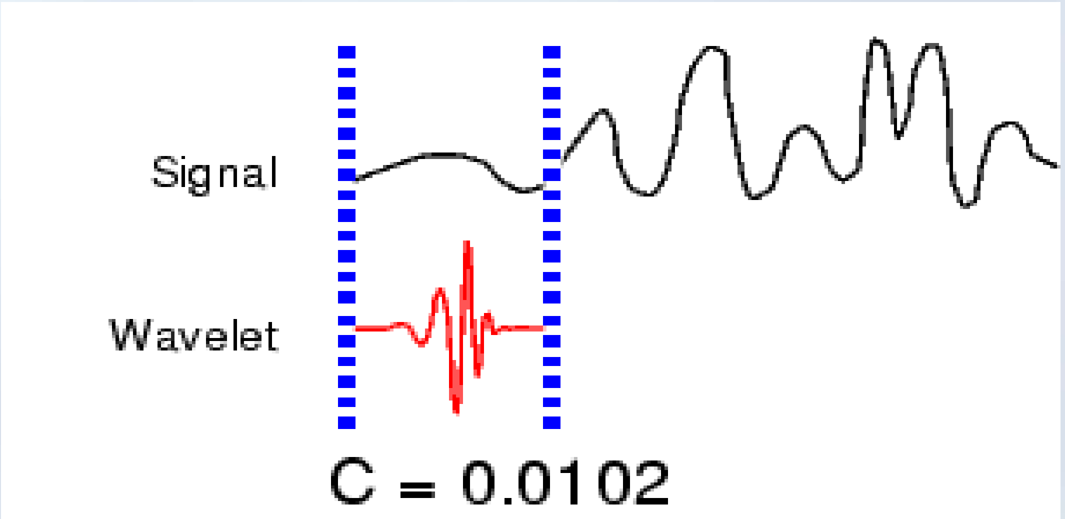
- Shift for wavelet function



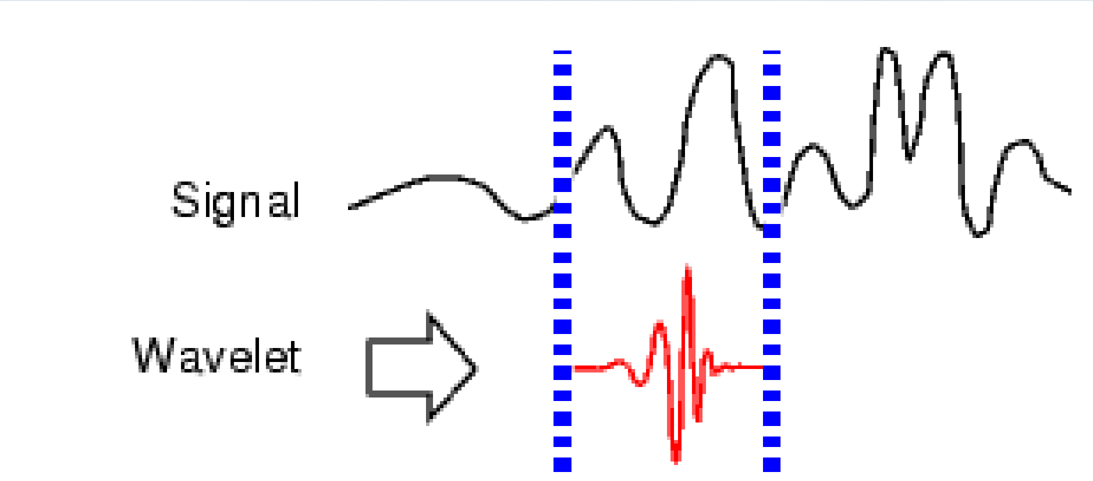


Steps to compute a continuous wavelet transform

- Take a wavelet and calculate its similarity to the original signal

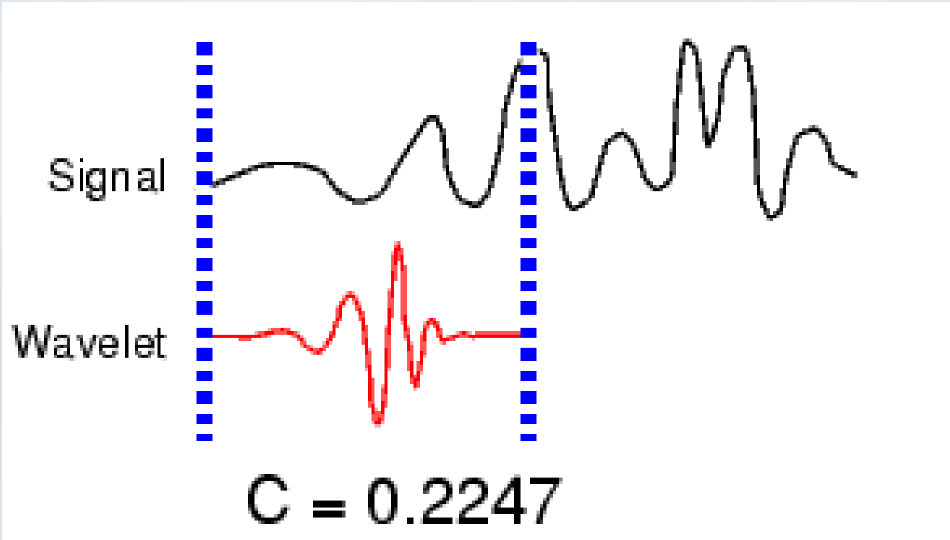


- Shift the wavelet and repeat

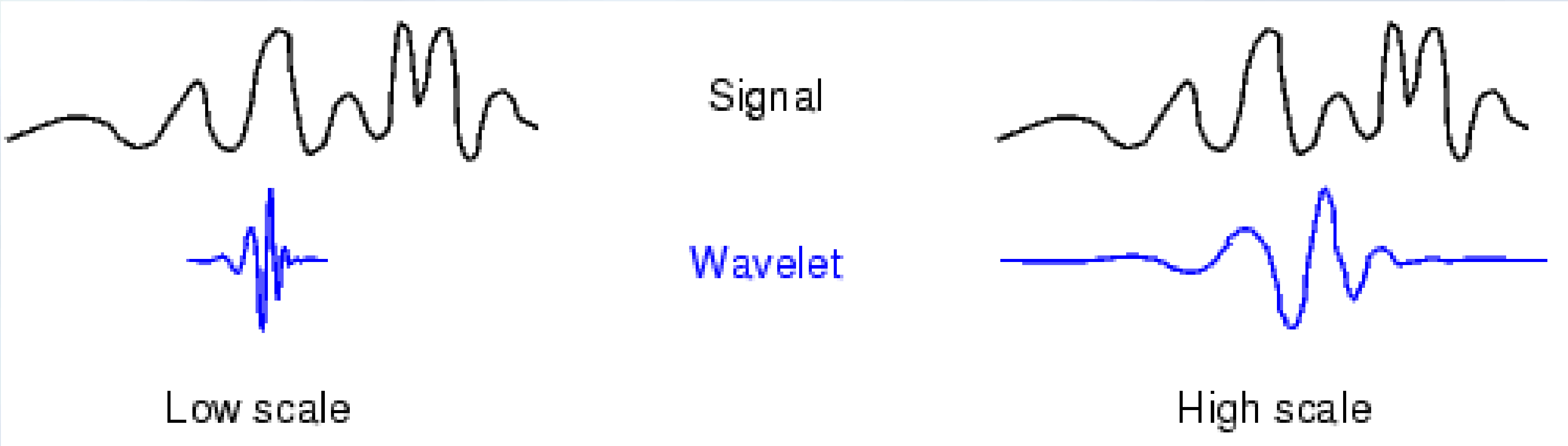


Steps to compute a continuous wavelet transform (2)

- Scale the wavelet and repeat



Scale and frequency



Rapid change
High frequency

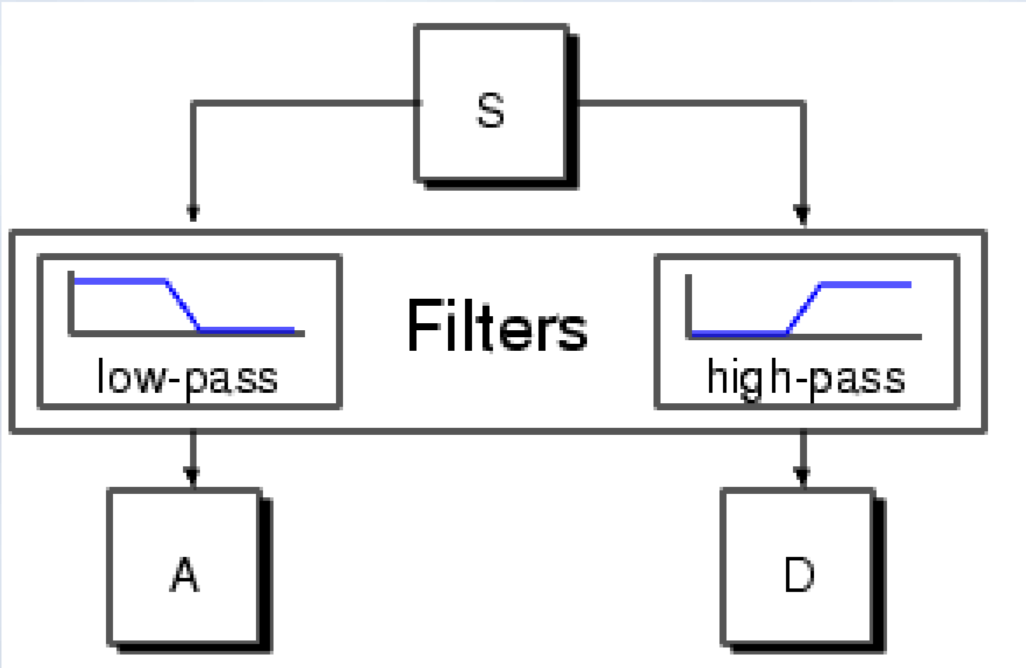
Slow change
Low frequency

Discrete wavelet transform

- Continuous wavelet transform: calculate wavelet coefficient at every possible scale and shift
- Discrete wavelet transform: choose scale and shift on powers of two (dyadic scale and shift)
  - Fast wavelet transform exist
  - Perfect reconstruction

Filtering structure for wavelet transform

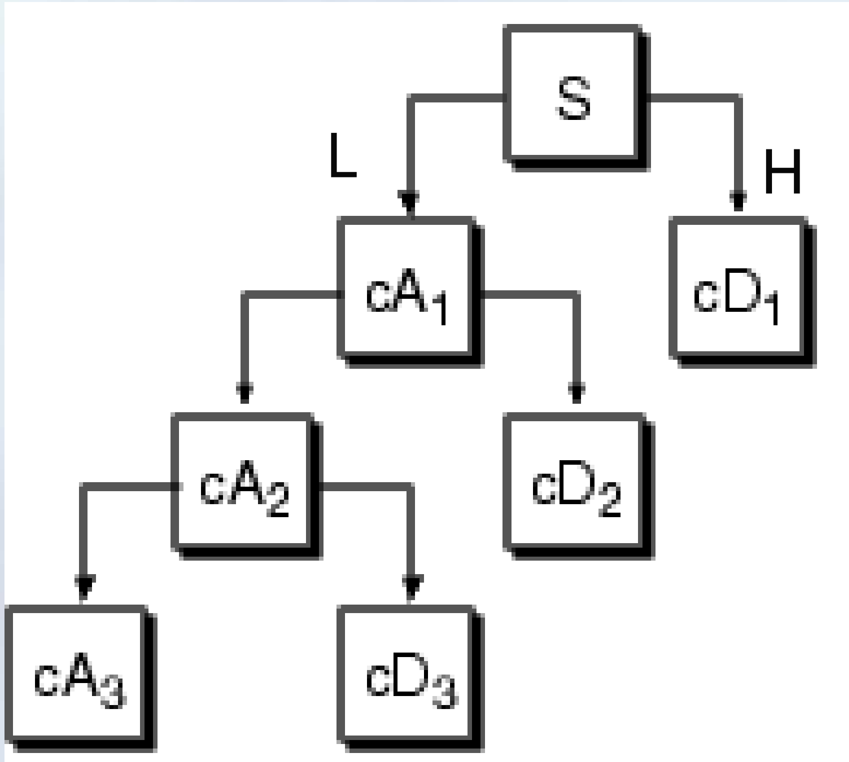
- S. Mallat derived the subband filtering structure for wavelet transform

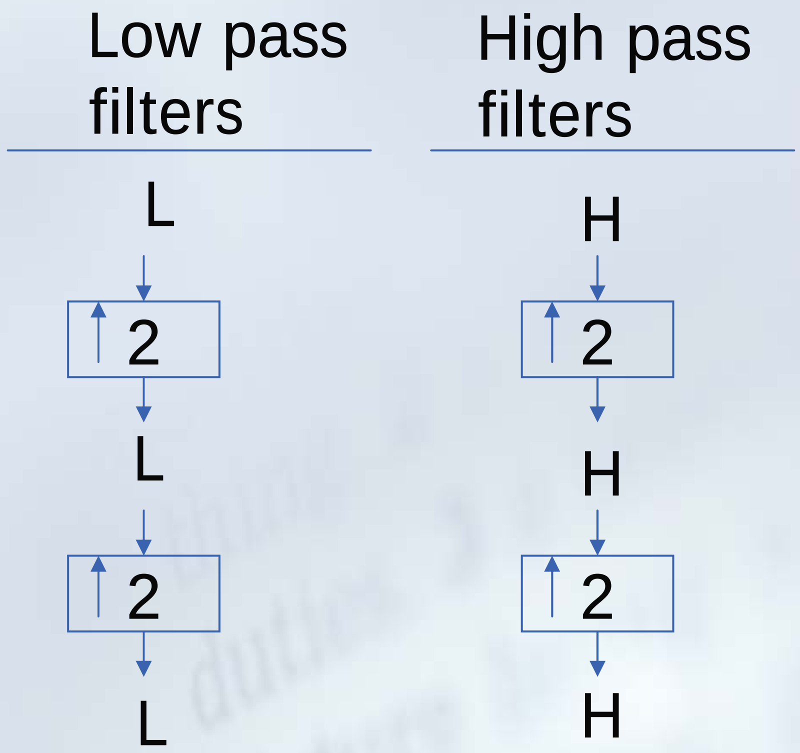


Approximation Detail

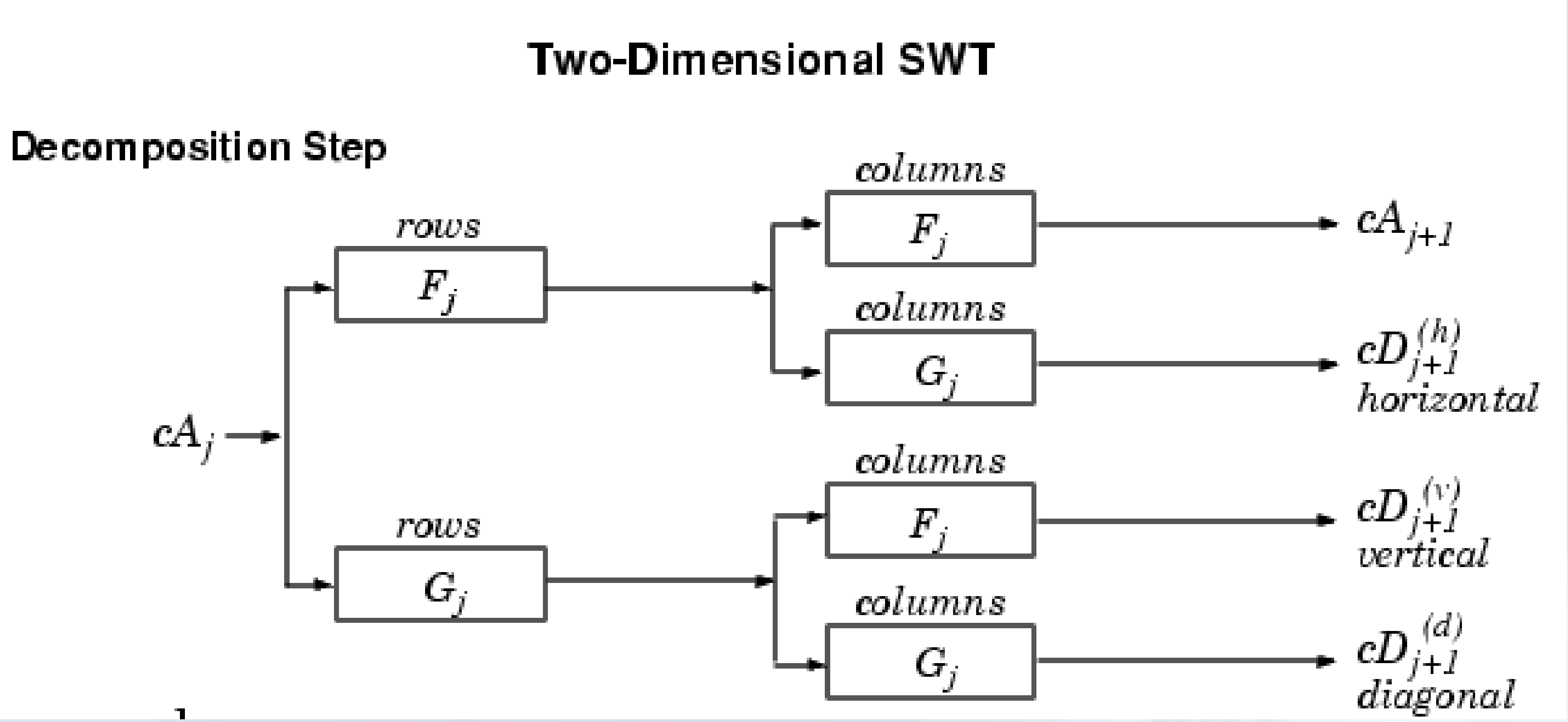
Multi-level decomposition

- Wavelet decomposition tree



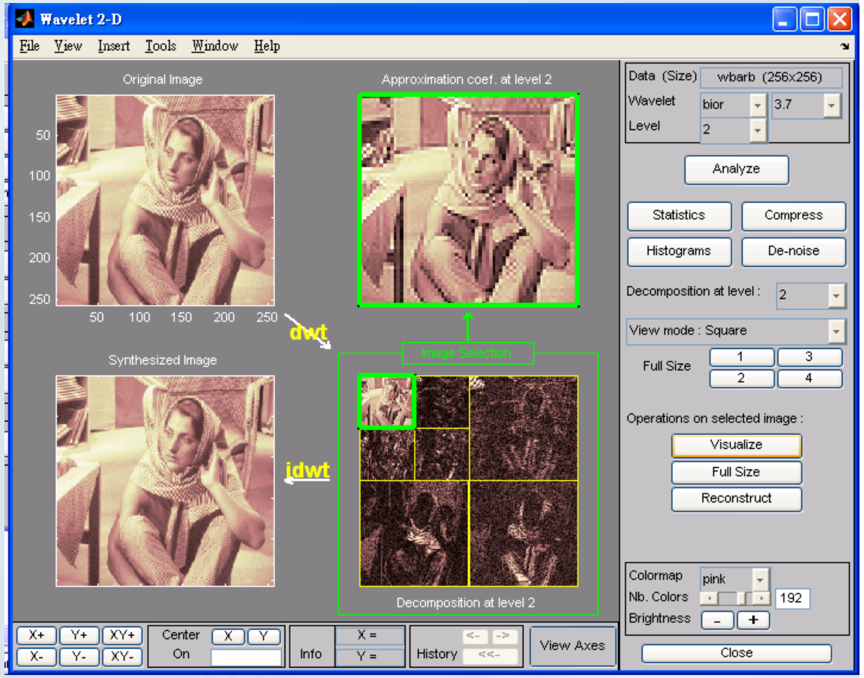


Two-dimensional wavelet transform



DWT using Matlab

- *wavemenu*
- Choose **wavelet 2-D**
- Load image -> toolbox/wavelet/wavedemo/wbarb.mat
- Bior3.7, level 2
- Square and tree mode



- **Subband coding**
  - An important multiresolution image analysis
  - An image is decomposed into a set of band-limited components- *subbands*
  - Each subband is generated by bandpass filtering

  - *Since the bandwith of subbands is smaller than that of the original image → the subbands can be downsampled without loss of information.*

- *Reconstruction of the original image is accomplished by upampling, filtering, and summing the subbands.*

- See Fig. 7.4.

- Analysis filters $h_0(n)$ and $h_1(n)$
- Synthesis filters $g_0(n)$ and $g_1(n)$

a
b

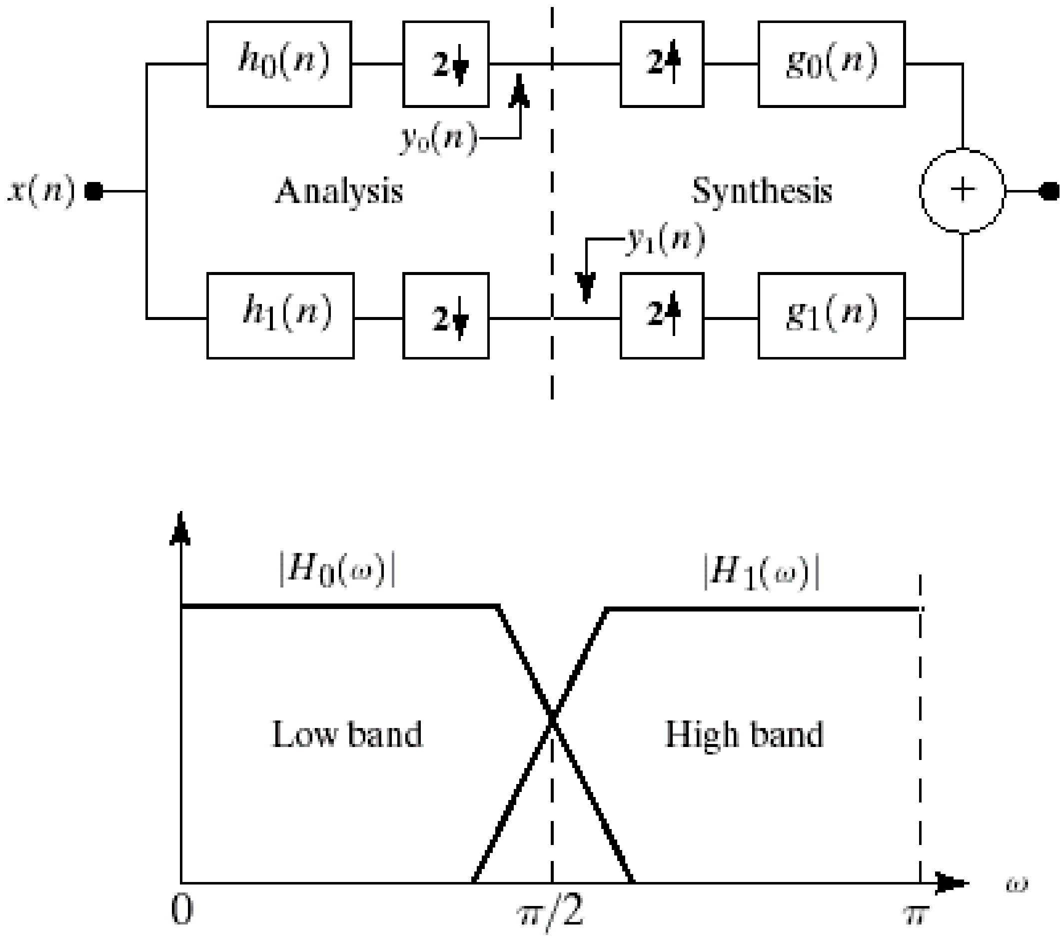


FIGURE 7.4 (a) A two-band filter bank for one-dimensional subband coding and decoding, and (b) its spectrum splitting properties.

Background

- *Perfect reconstruction filters*
  - Quadrature mirror filters (QMF)
  - Conjugate quadrature filters (CQF)
  - Orthonormal filters (used for wavelet transformation)

Filter	QMF	CQF	Orthonormal
$H_0(z)$	$H_0^2(z) - H_0^2(-z) = 2$	$H_0(z)H_0(z^{-1}) +$ $H_0^2(-z)H_0(-z^{-1}) = 2$	$G_0(z^{-1})$
$H_1(z)$	$H_0(-z)$	$z^{-1}H_0(-z^{-1})$	$G_1(z^{-1})$
$G_0(z)$	$H_0(z)$	$H_0(z^{-1})$	$G_0(z)G_0(z^{-1}) +$ $G_0(-z)G_0(-z^{-1}) = 2$
$G_1(z)$	$-H_0(-z)$	$zH_0(-z)$	$-z^{-2K+1}G_0(-z^{-1})$

TABLE 7.1 Perfect reconstruction filter families.

2D subband filtering

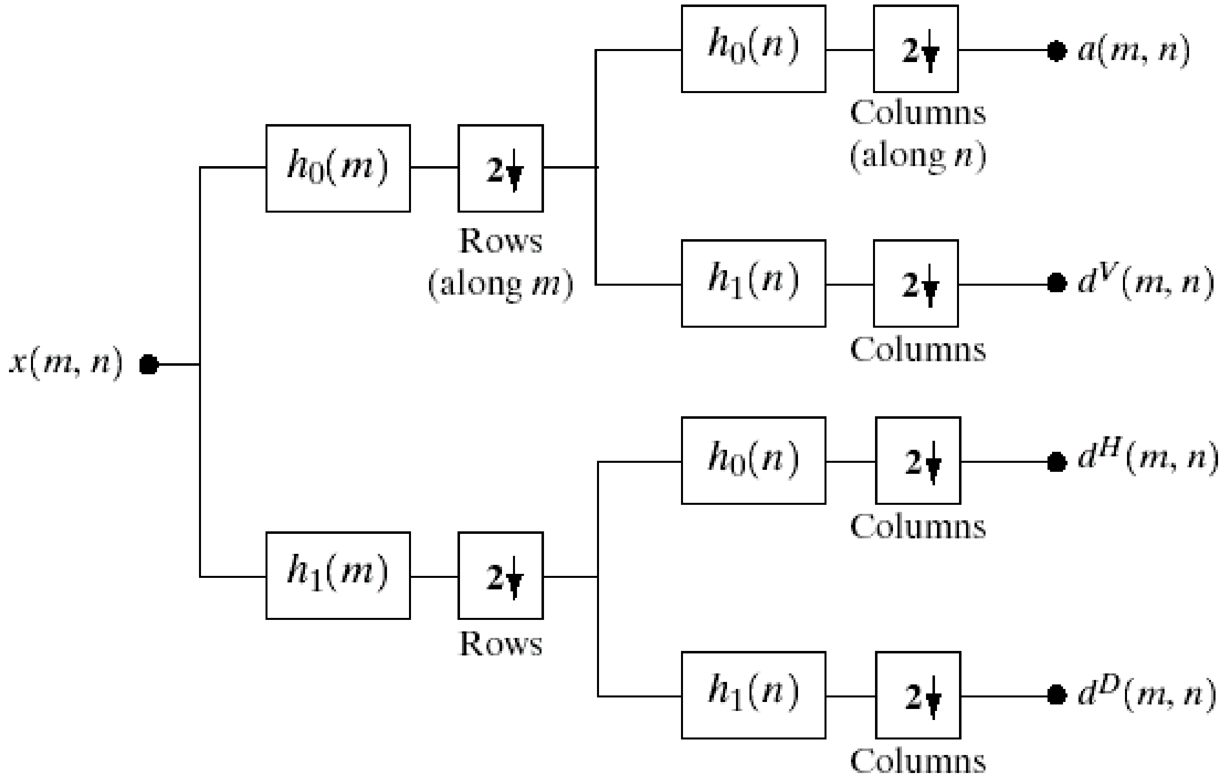


FIGURE 7.5 A two-dimensional, four-band filter bank for subband image coding.

- *Example 7.2*
  - 8-tap orthonormal filters ; Results in Fig. 7.7

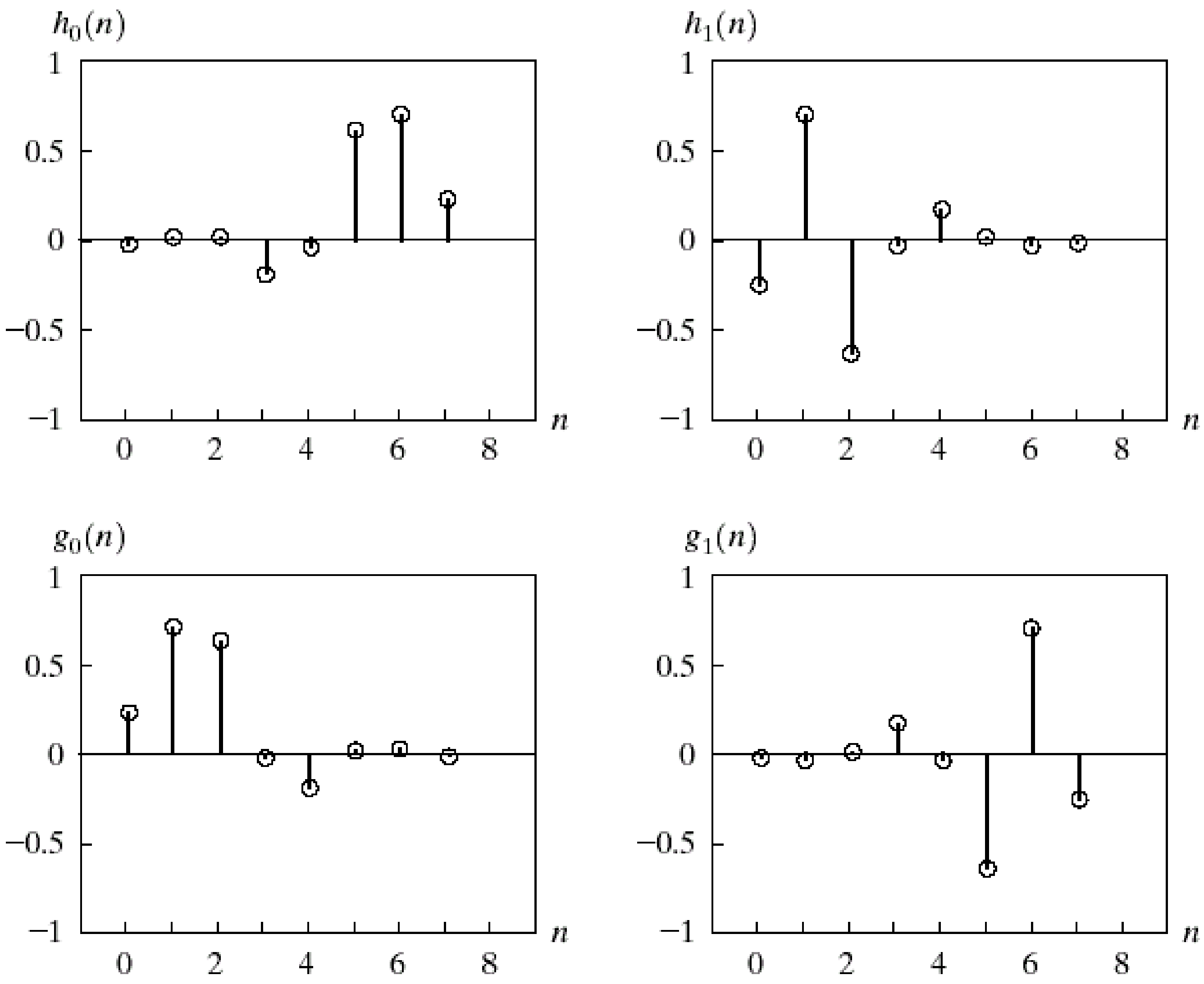


FIGURE 7.6 The impulse responses of four 8-tap Daubechies orthonormal filters.

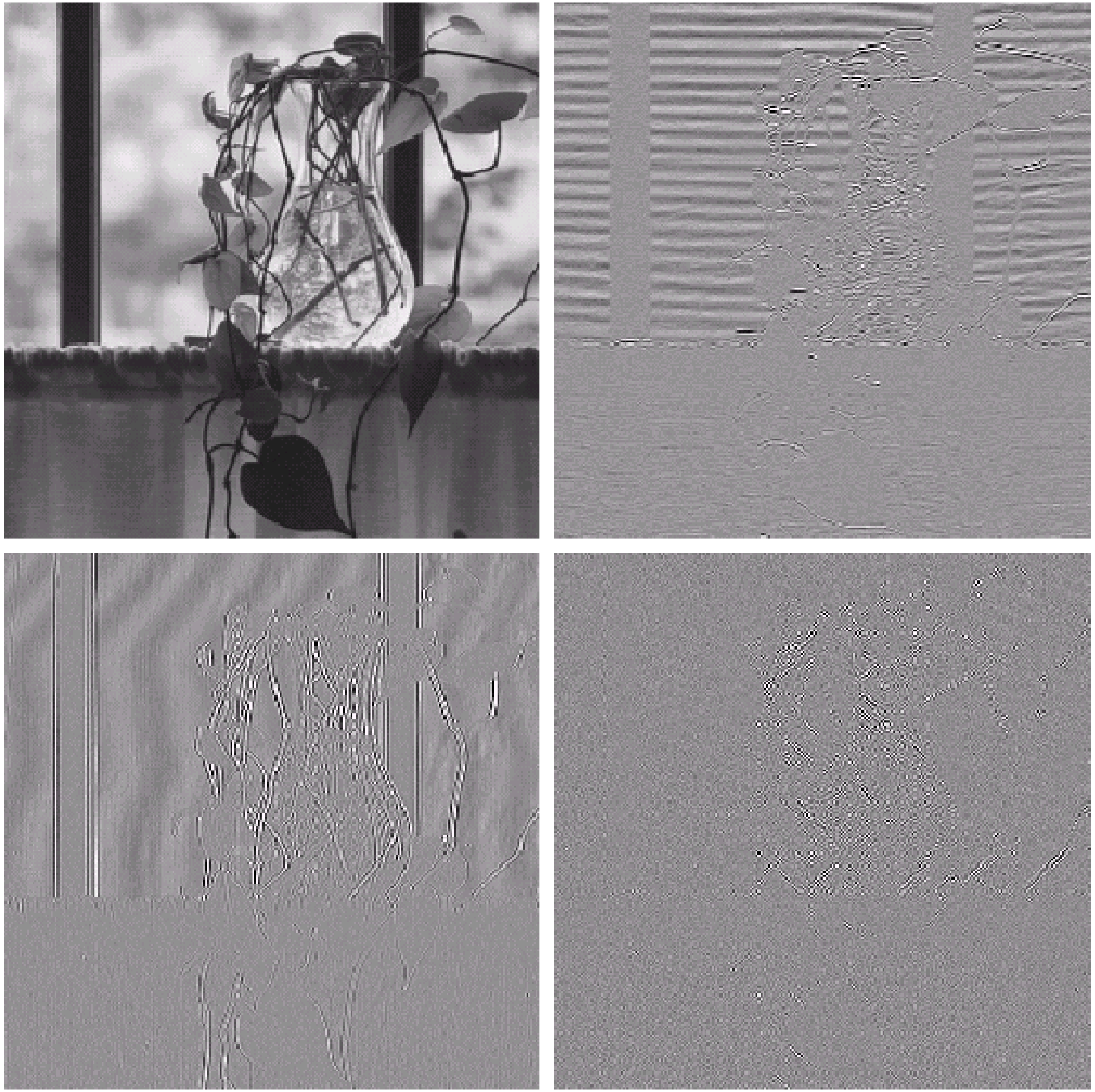


FIGURE 7.7 A four-band split of the vase in Fig. 7.1 using the subband coding system of Fig. 7.5.

Background

- *Haar Transform*
  - Oldest and simplest orthonormal wavelets
  - Transformation matrix in page 362.

  - Example 7.3
    - Discrete wavelet transformation using Haar basis.
    - In Fig. 7.8

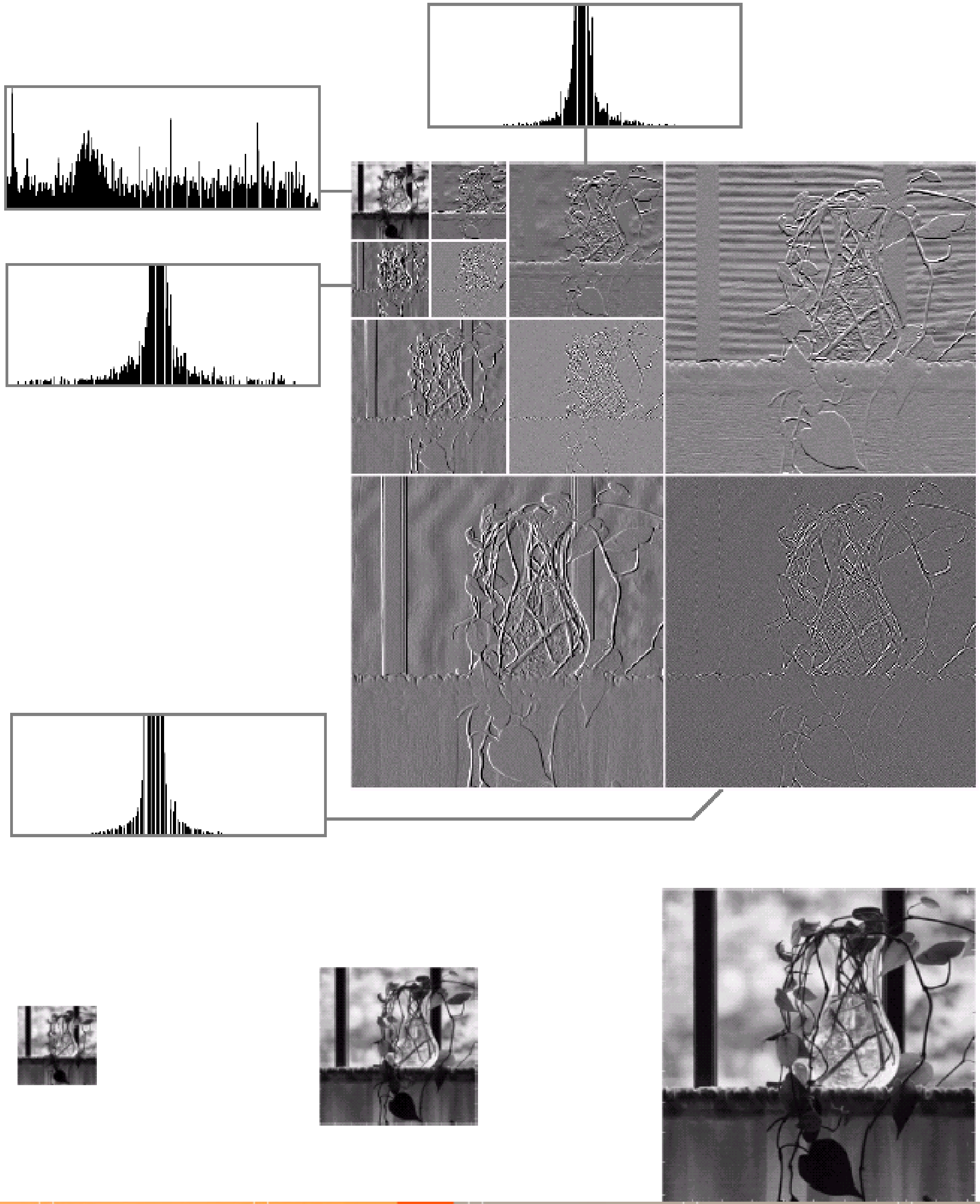
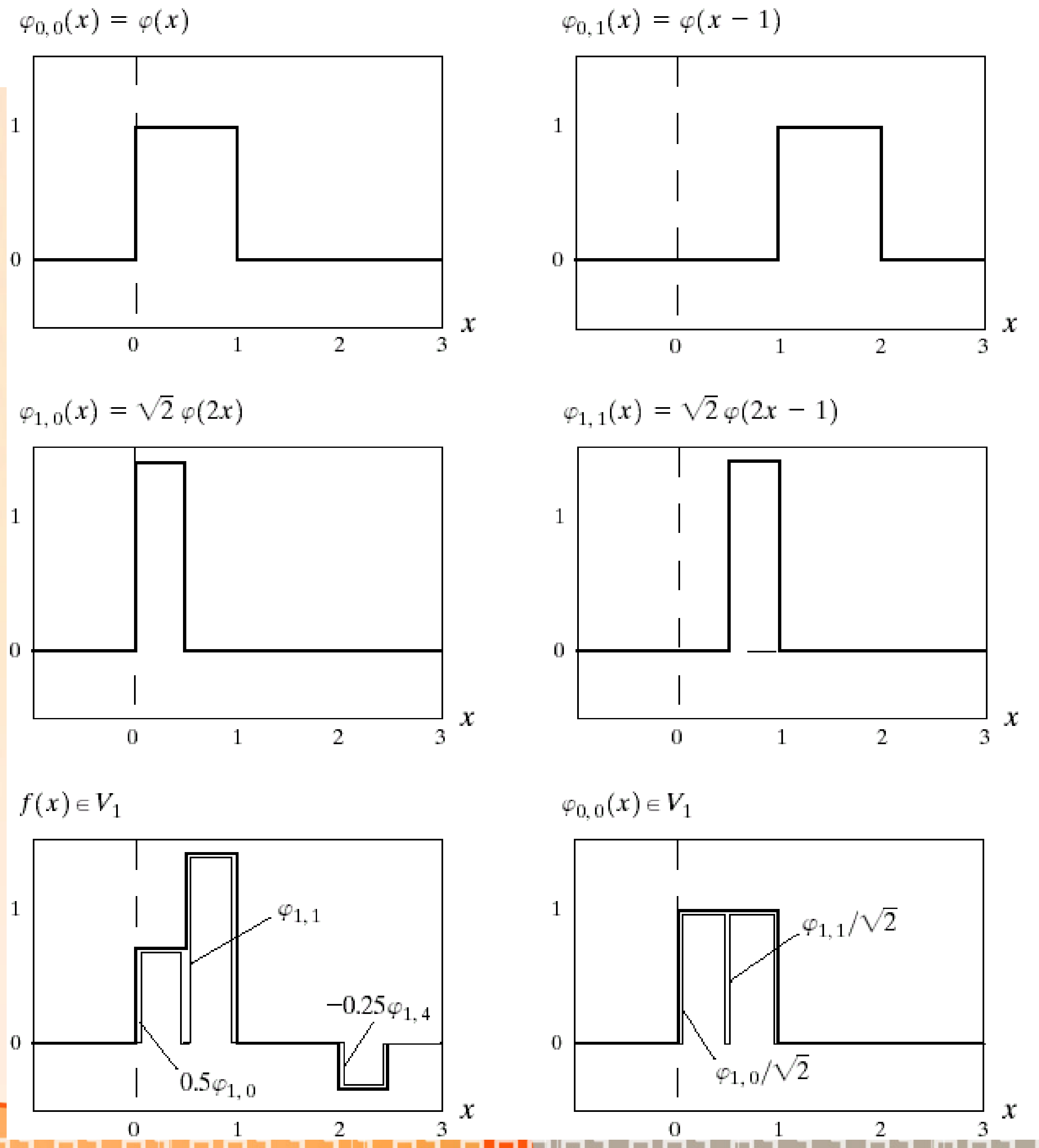


FIGURE 7.8 (a) A discrete wavelet transform using Haar basis functions. Its local histogram variations are also shown; (b)–(d) Several different approximations (64 × 64, 128 × 128, and 256 × 256) that can be obtained from (a).

Multiresolution Expansions

- *Series expansions*
  - Expansion coefficients
  - Expansion functions.
    - If the expansion is unique - Basis functions
- Scaling function
  - Binary scalings
    - Eq. 7.2-10
  - Example 7.4
    - The Haar scaling function

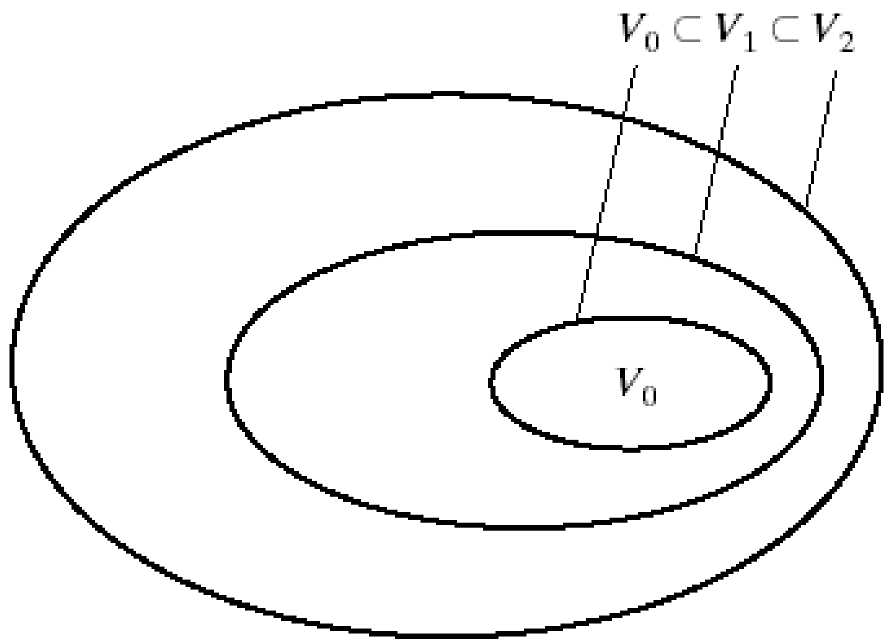


a b
c d
e f

FIGURE 7.9 Haar scaling functions in V_0 in V_1.

Subapaces containing high-resolution function must also contain all low resolution functions

FIGURE 7.10 The nested function spaces spanned by a scaling function.



Wavelet Functions

$$\psi_{j,k}(x) = 2^{\frac{j}{2}} \psi\left(2^j x - k\right) \qquad (7.2-19)$$

$$W_j = \underset{k}{Span}\{\psi_{j,k}(x)\} \qquad (7.2-20)$$

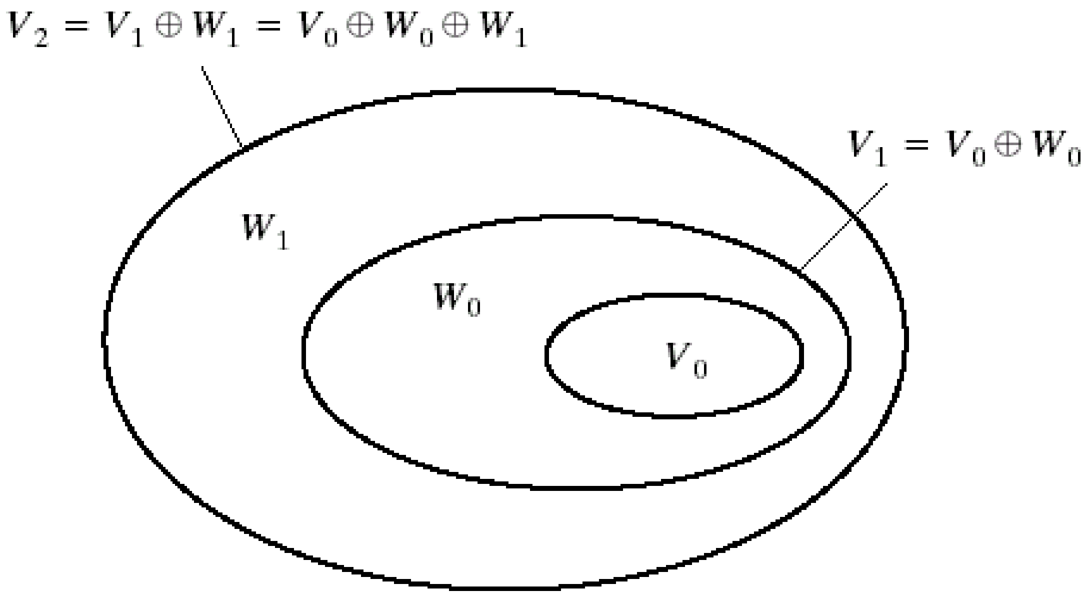


FIGURE 7.11 The relationship between scaling and wavelet function spaces.

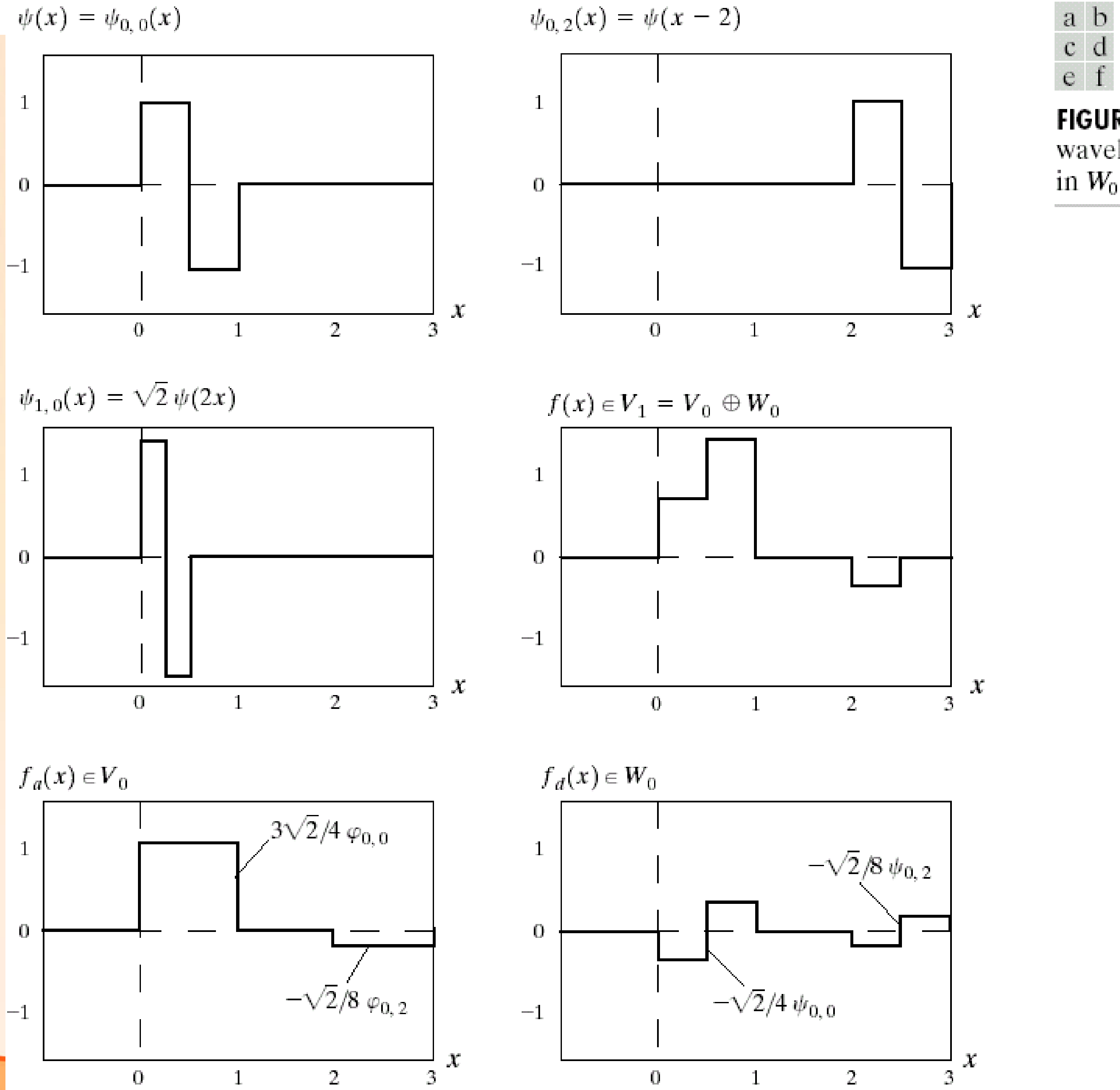


FIGURE 7.12 Haar wavelet functions in W_0 and W_1.

7.3 WT in 1-D

- *Wavelet Series expansions*
  - Example 7.7
    - The Haar Wavelet Series expansion of $y = x^2$
- *Discrete Wavelet Transform*
  - *Eq. 7.3-5 ~7.3-7*
- *Continuous Wavelet Transform*
  - *Eq. 7.3-8 ~7.3-11*

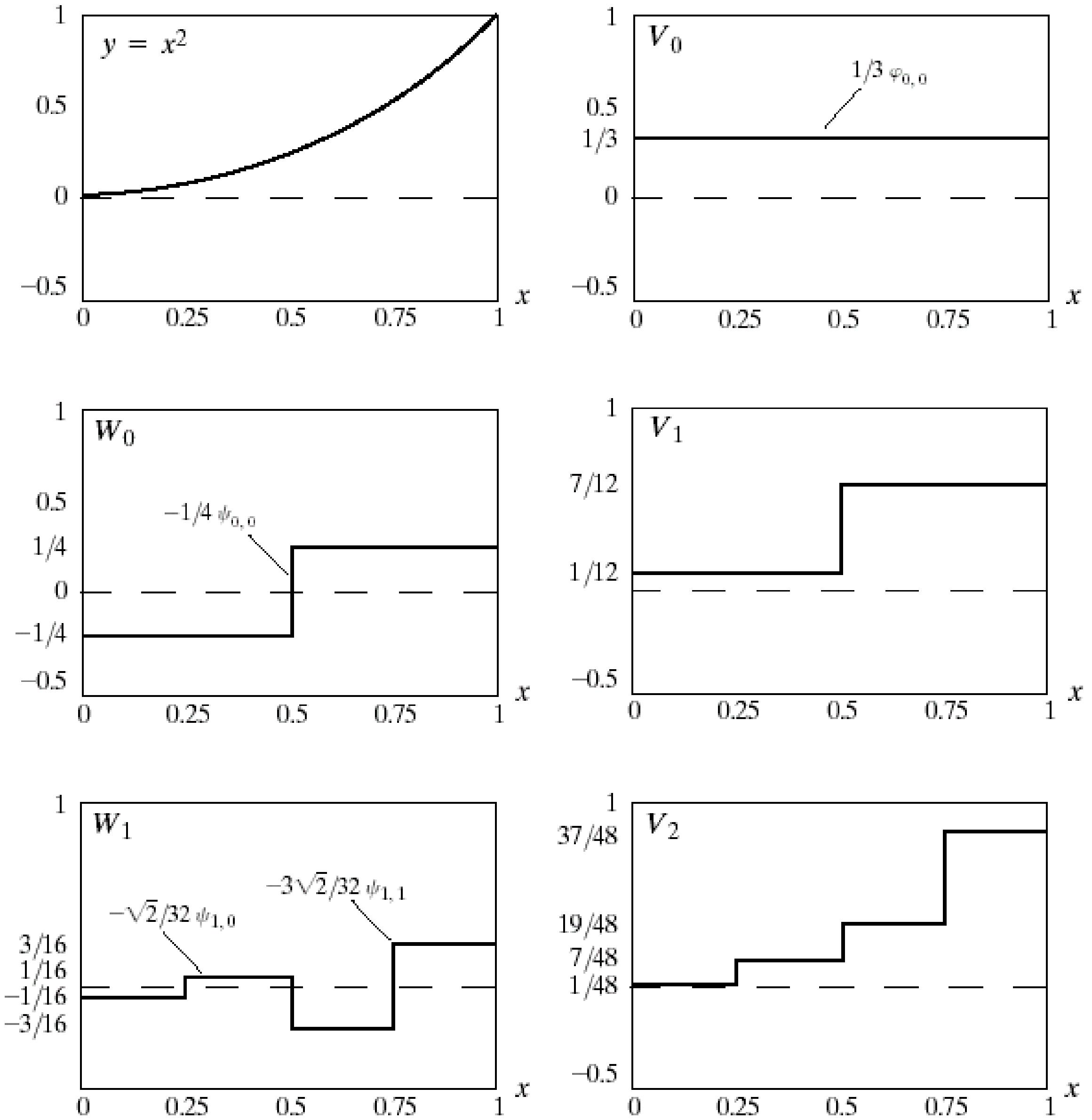
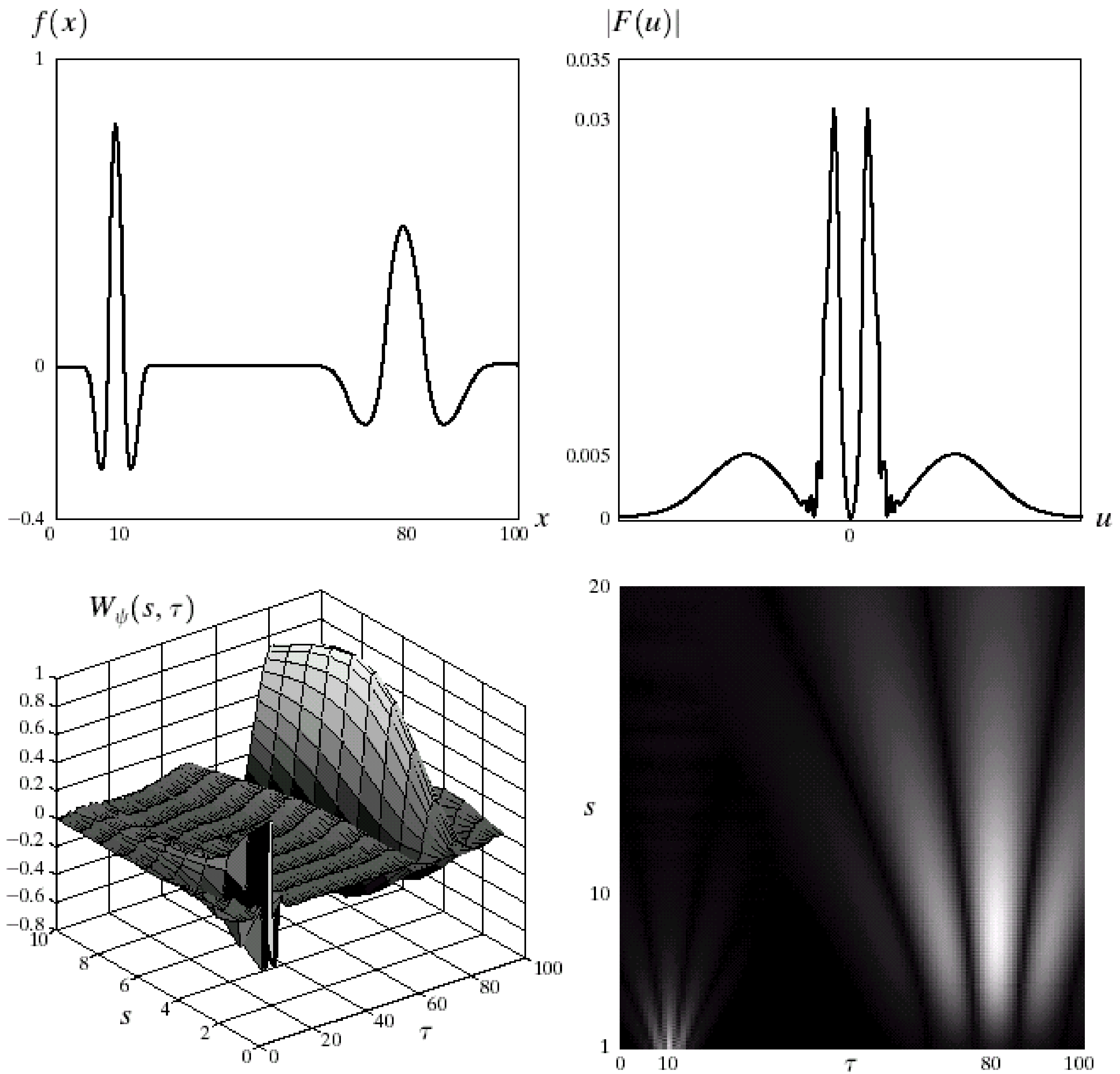


FIGURE 7.13 A wavelet series expansion of $y = x^2$ using Haar wavelets.

a b
c d

FIGURE 7.14 The continuous wavelet transform (c and d) and Fourier spectrum (b) of a continuous one-dimensional function (a).



The Fast WT

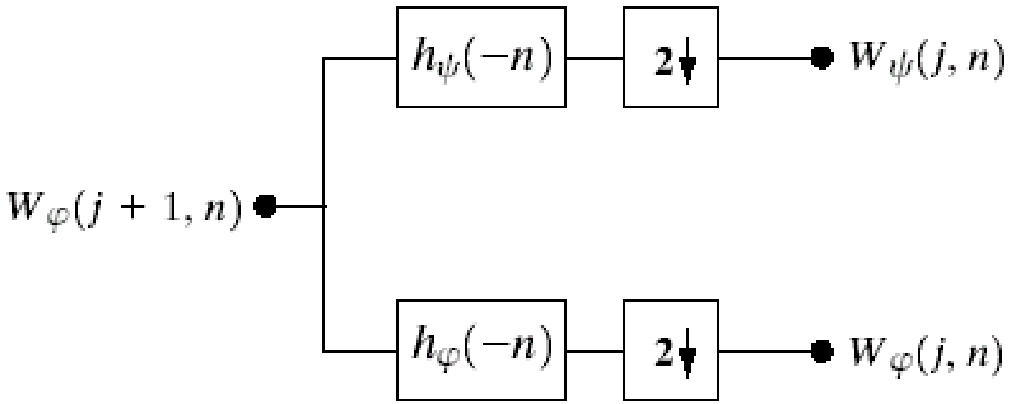
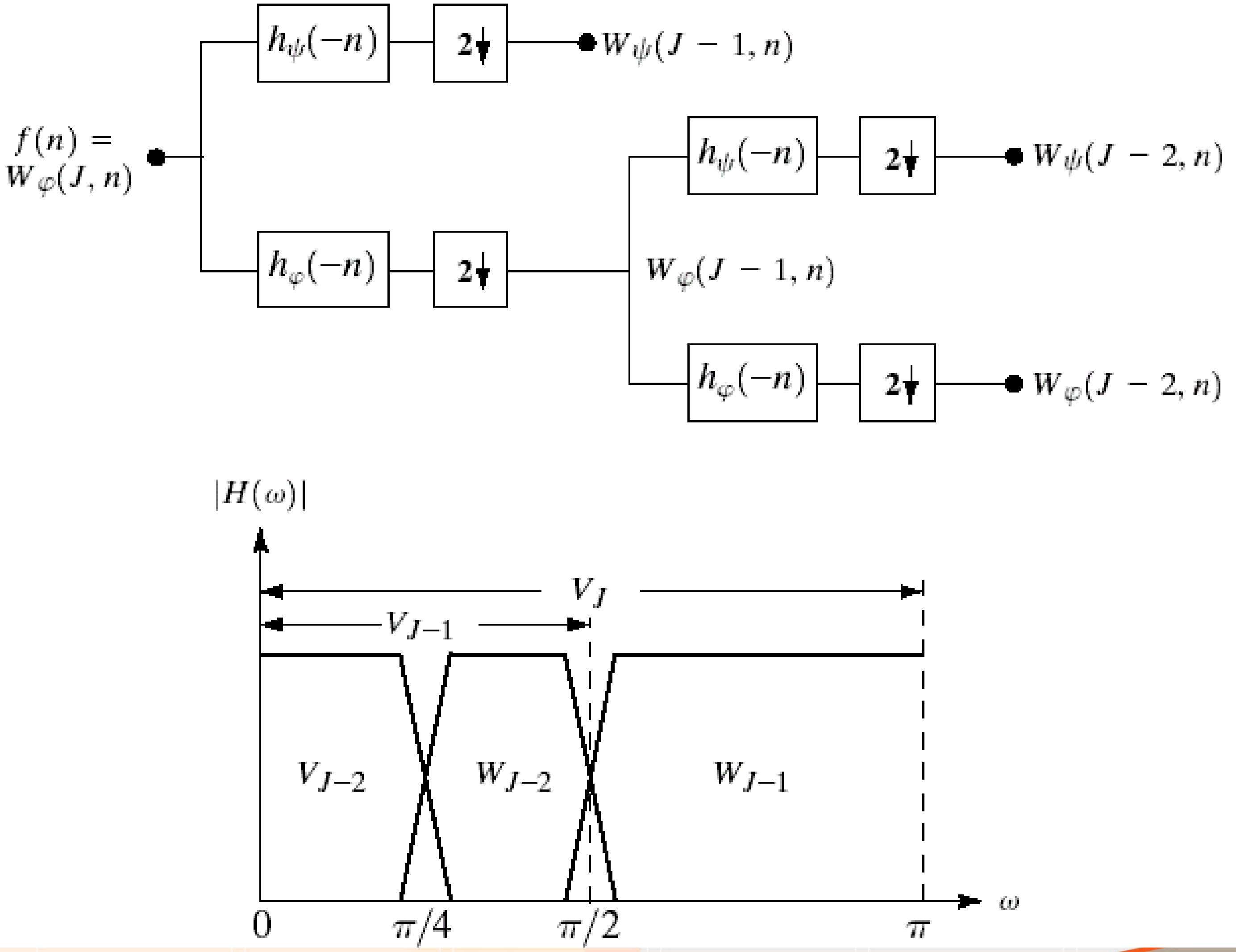


FIGURE 7.15 An FWT analysis bank.



a
b

FIGURE 7.16
(a) A two-stage or two-scale FWT analysis bank and (b) its frequency splitting characteristics.

Example 7-10 1-D FWT

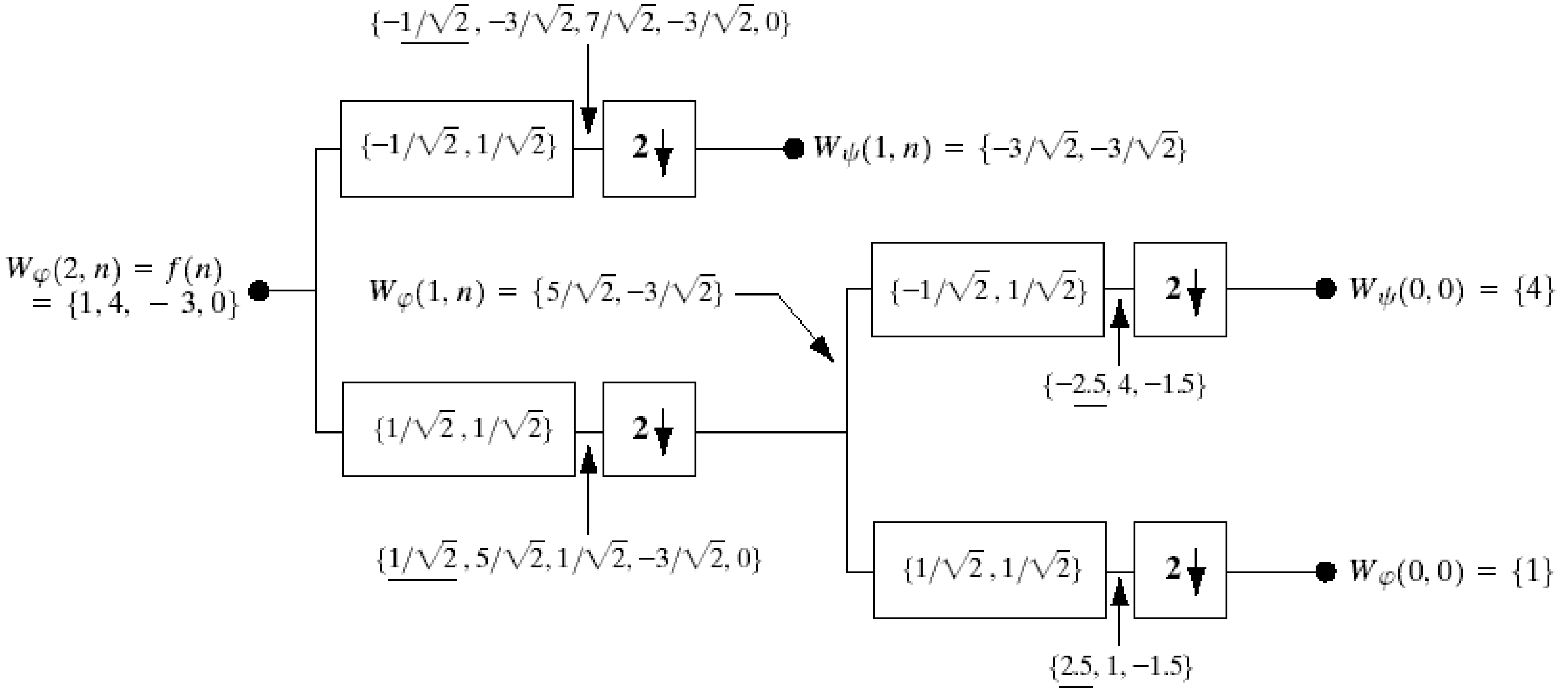


FIGURE 7.17 Computing a two-scale fast wavelet transform of sequence {1, 4, −3, 0} using Haar scaling and wavelet vectors.

FWT^{-1}

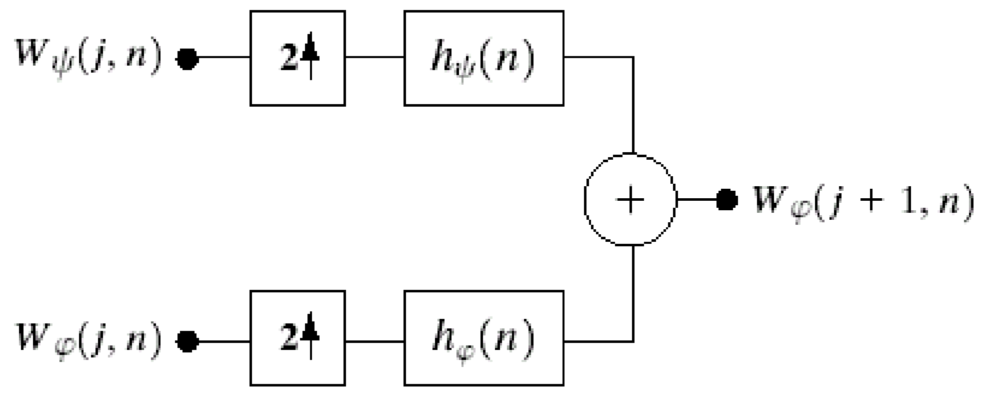
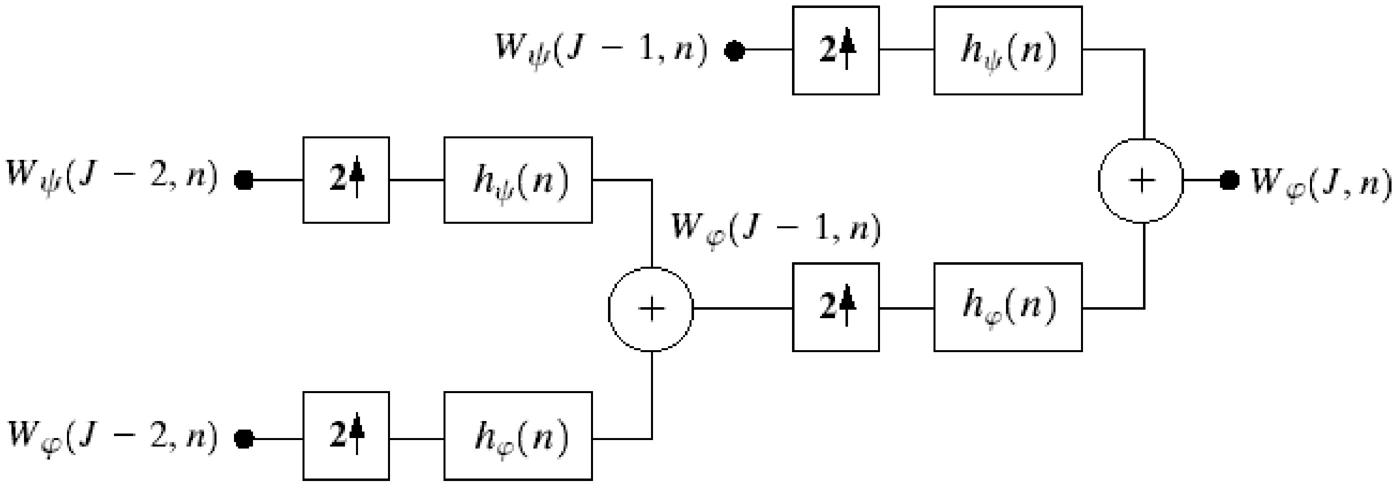


FIGURE 7.18 The FWT^{-1} synthesis filter bank.

Two scale FWT-1

FIGURE 7.19 A two-stage or two-scale FWT^{-1} synthesis bank.



Example 7-10 1-D FWT-1

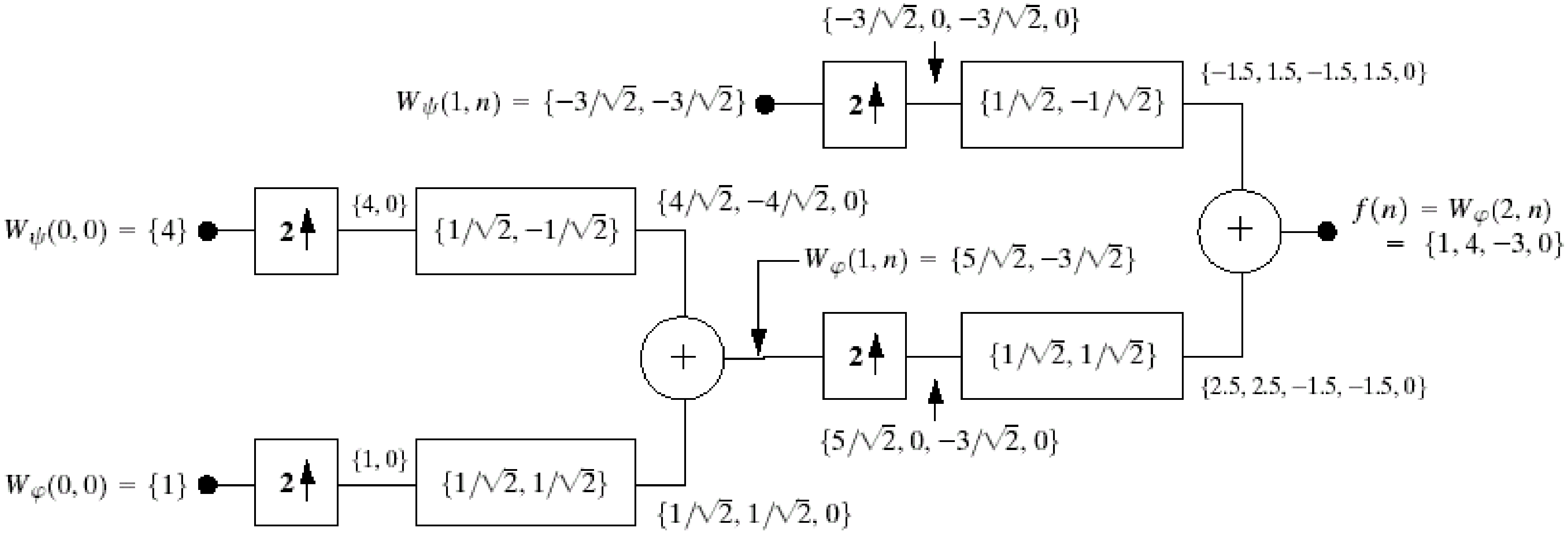


FIGURE 7.20 Computing a two-scale inverse fast wavelet transform of sequence $\{1, 4, -1.5\sqrt{2}, -1.5\sqrt{2}\}$ with Haar scaling and wavelet vectors.

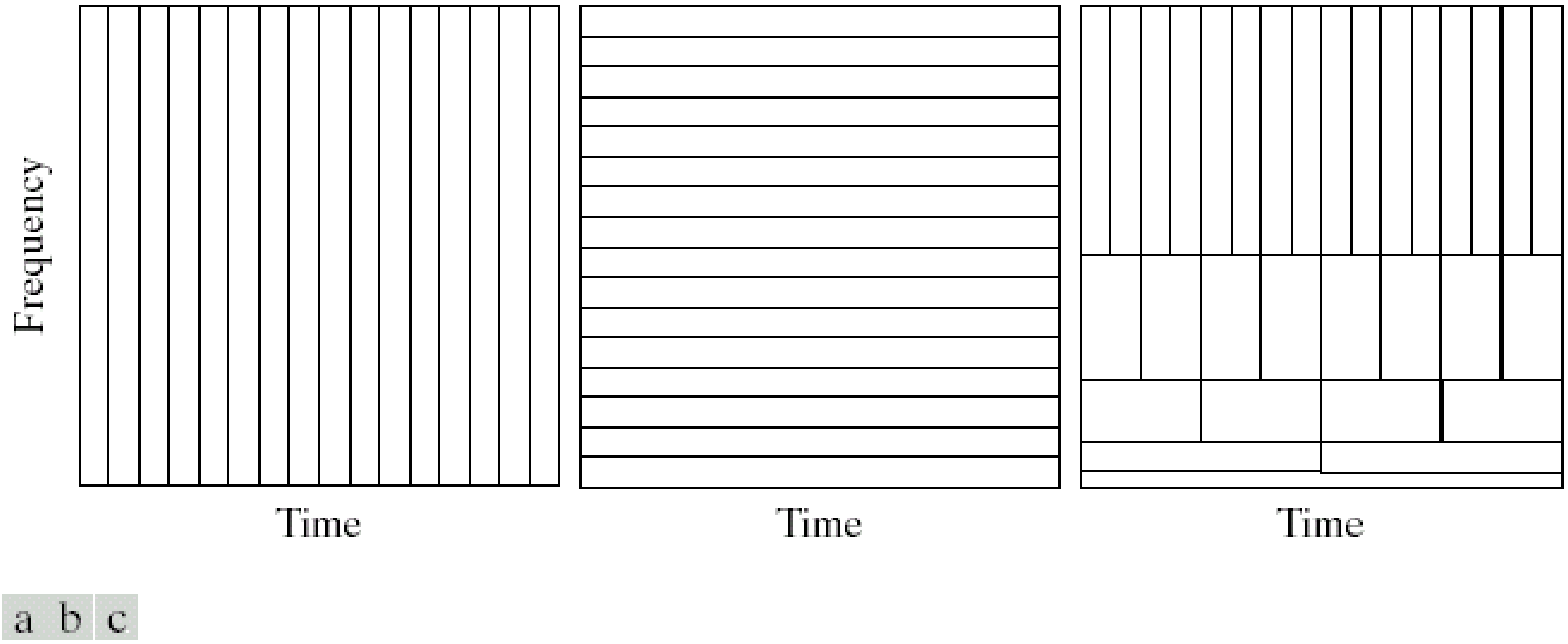
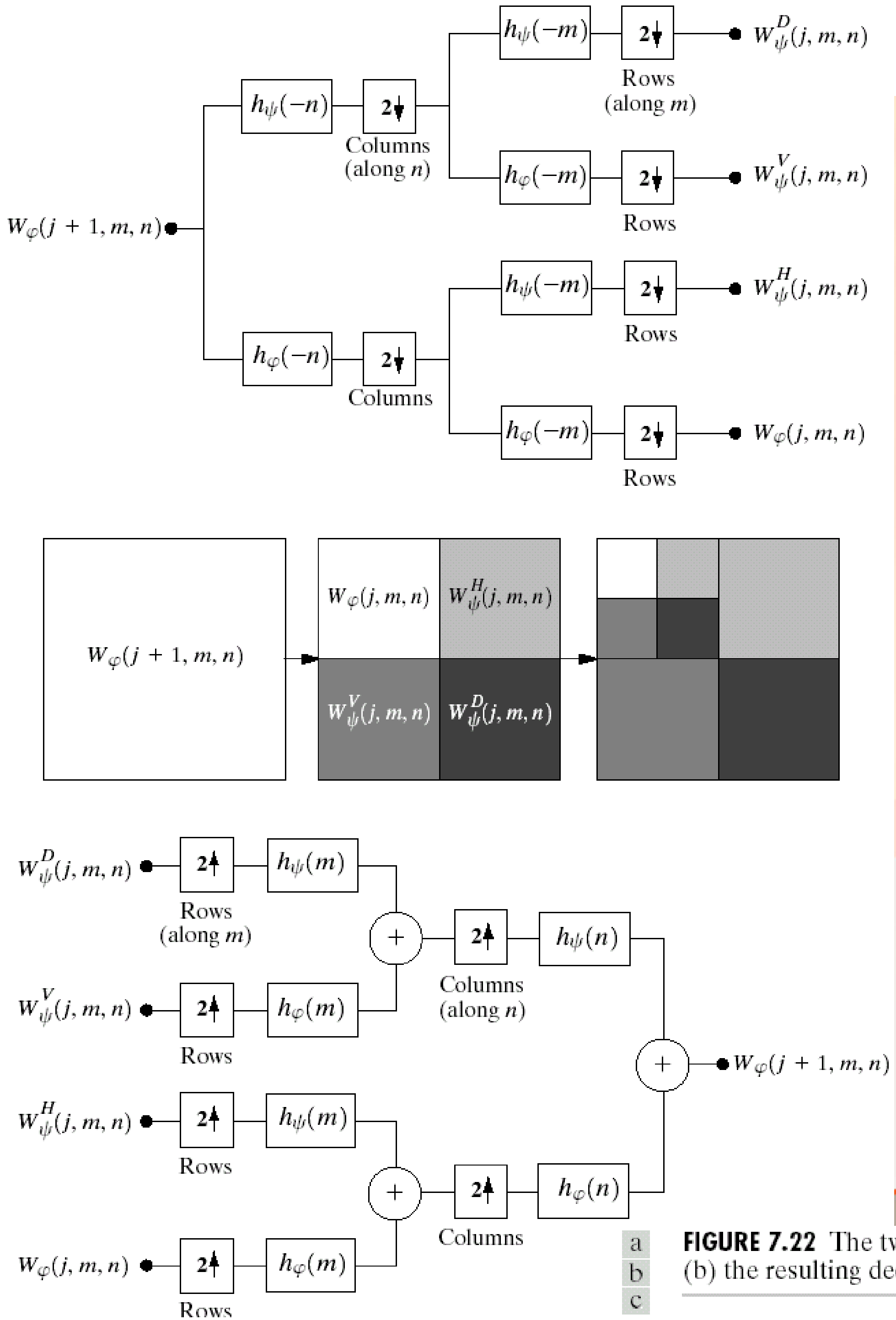


FIGURE 7.21 Time-frequency tilings for (a) sampled data, (b) FFT, and (c) FWT basis functions.

The 2-D WT



a
b
c

FIGURE 7.22 The two-dimensional fast wavelet transform: (a) the analysis filter bank; (b) the resulting decomposition; and (c) the synthesis filter bank.

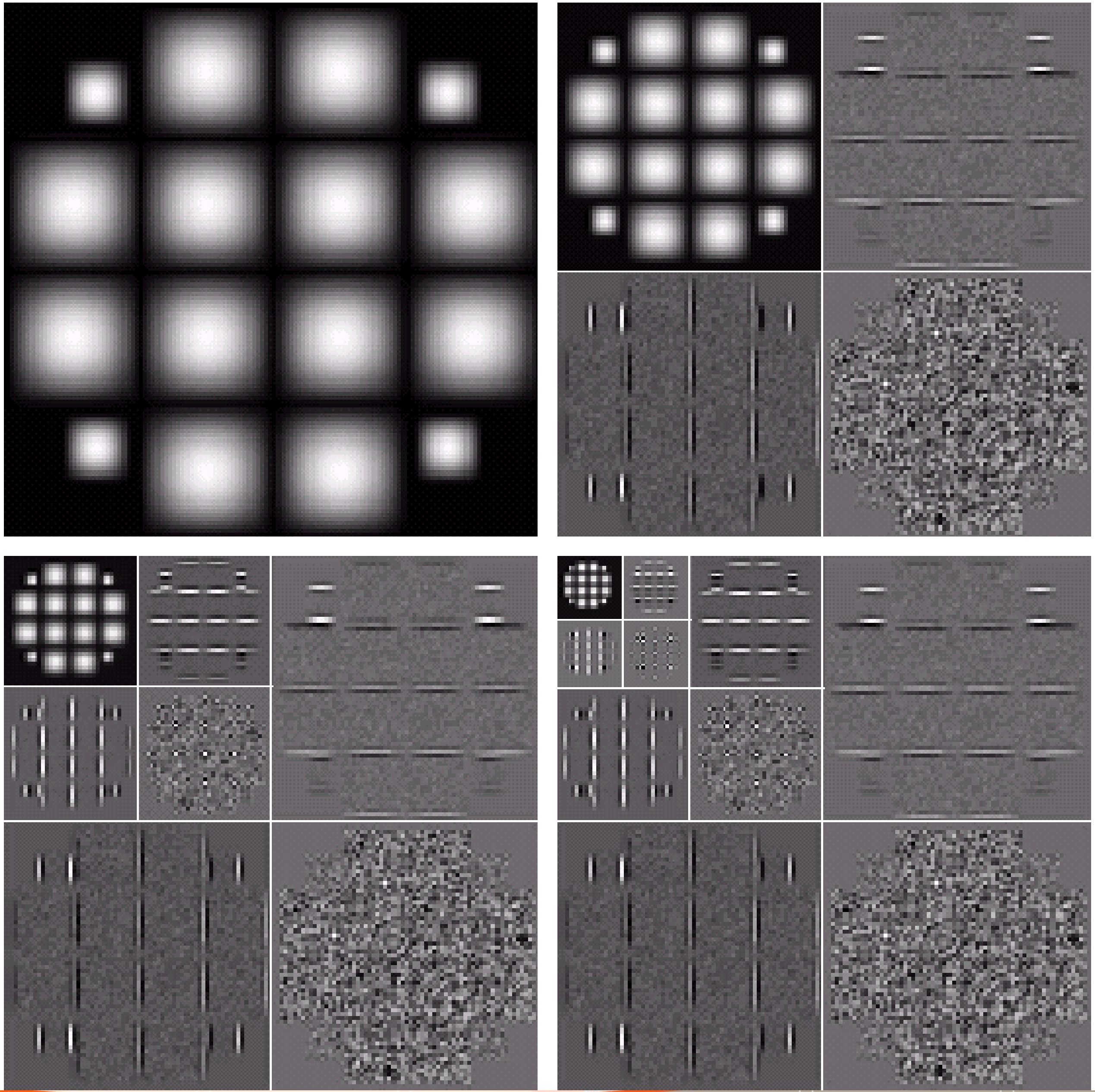


FIGURE 7.23 A three-scale FWT.

a b
c d
e f
g

FIGURE 7.24 Fourth-order symlets: (a)–(b) decomposition filters; (c)–(d) reconstruction filters; (e) the one-dimensional wavelet; (f) the one-dimensional scaling function; and (g) one of three two-dimensional wavelets, $\psi^H(x, y)$.

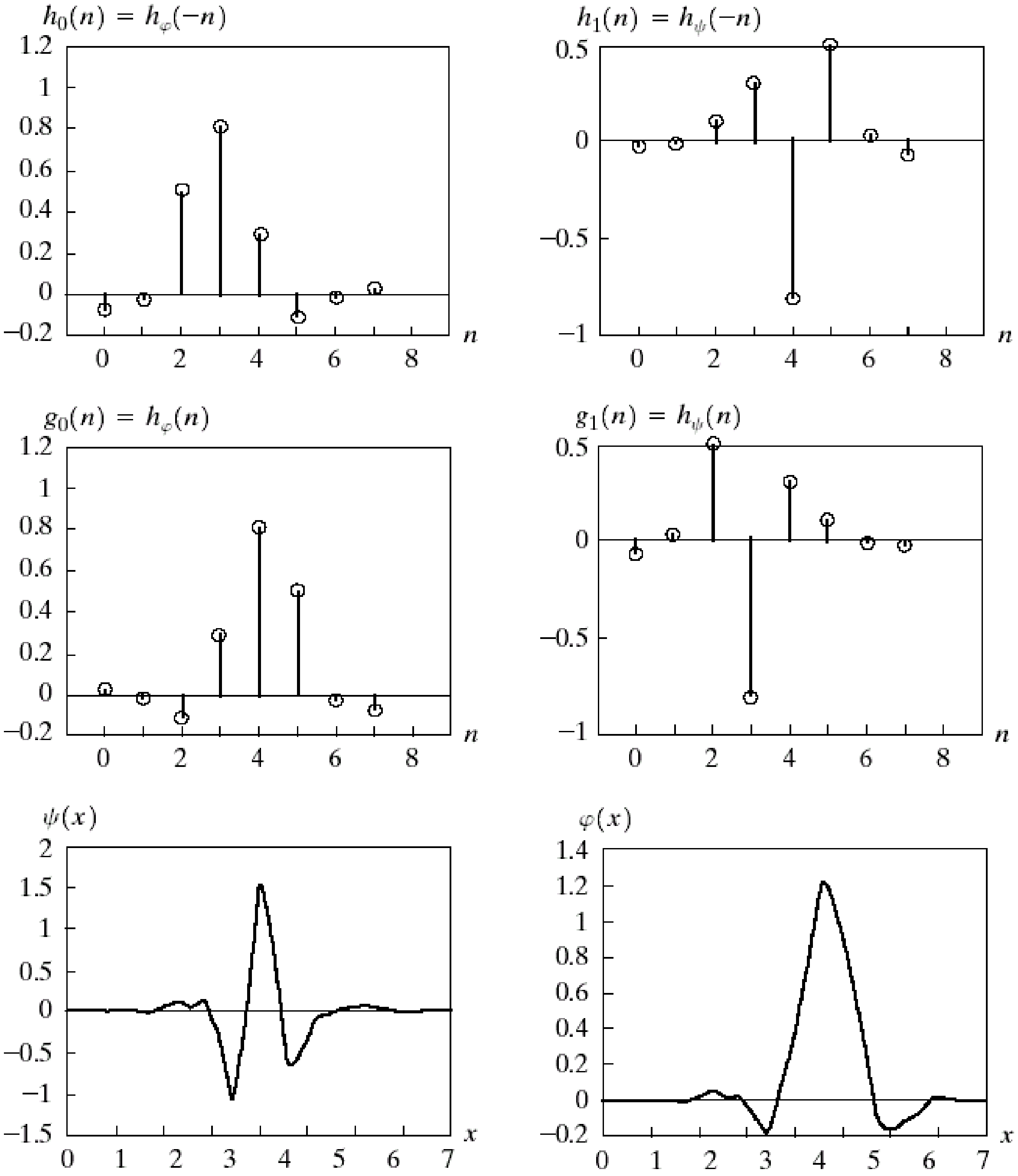
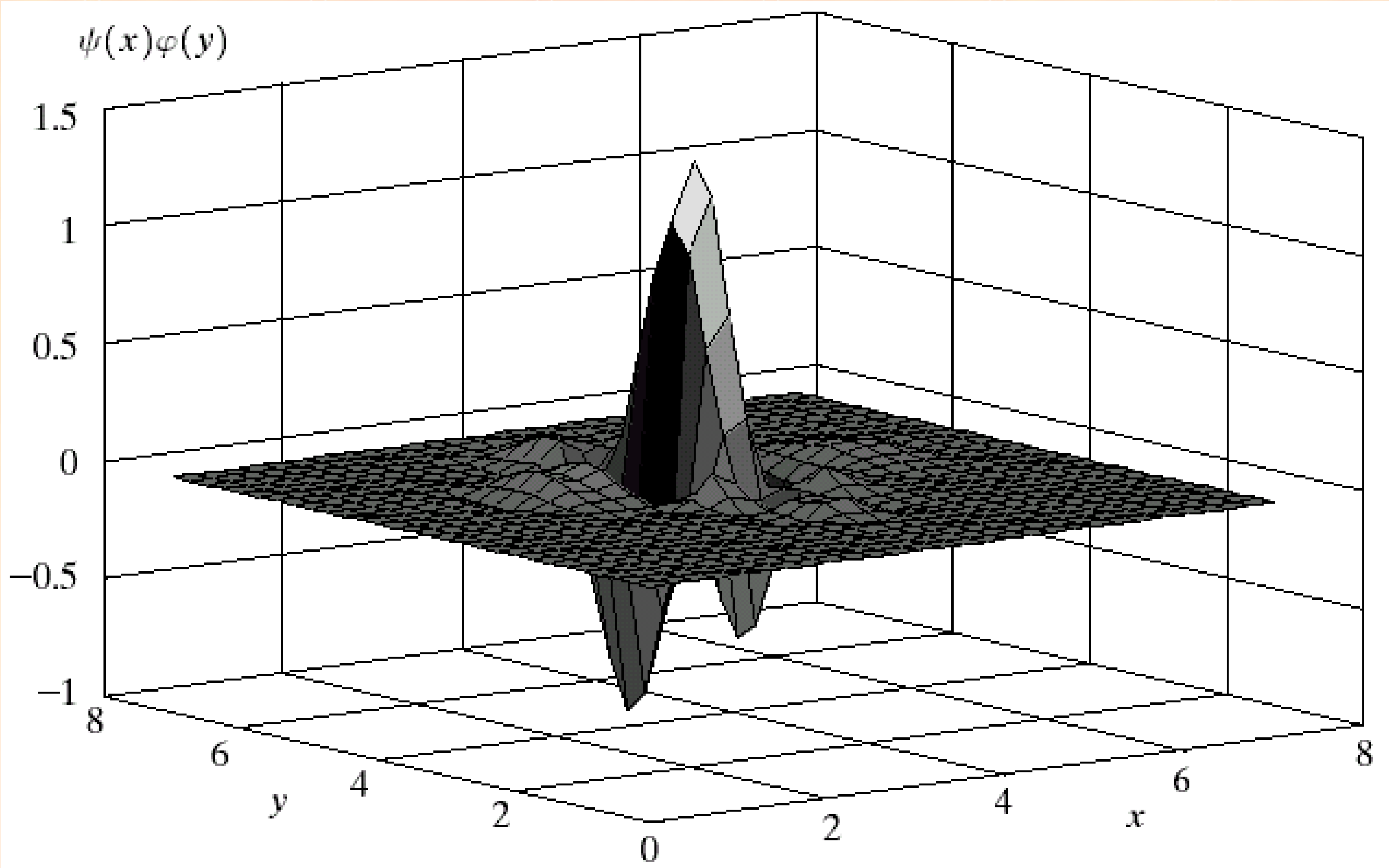
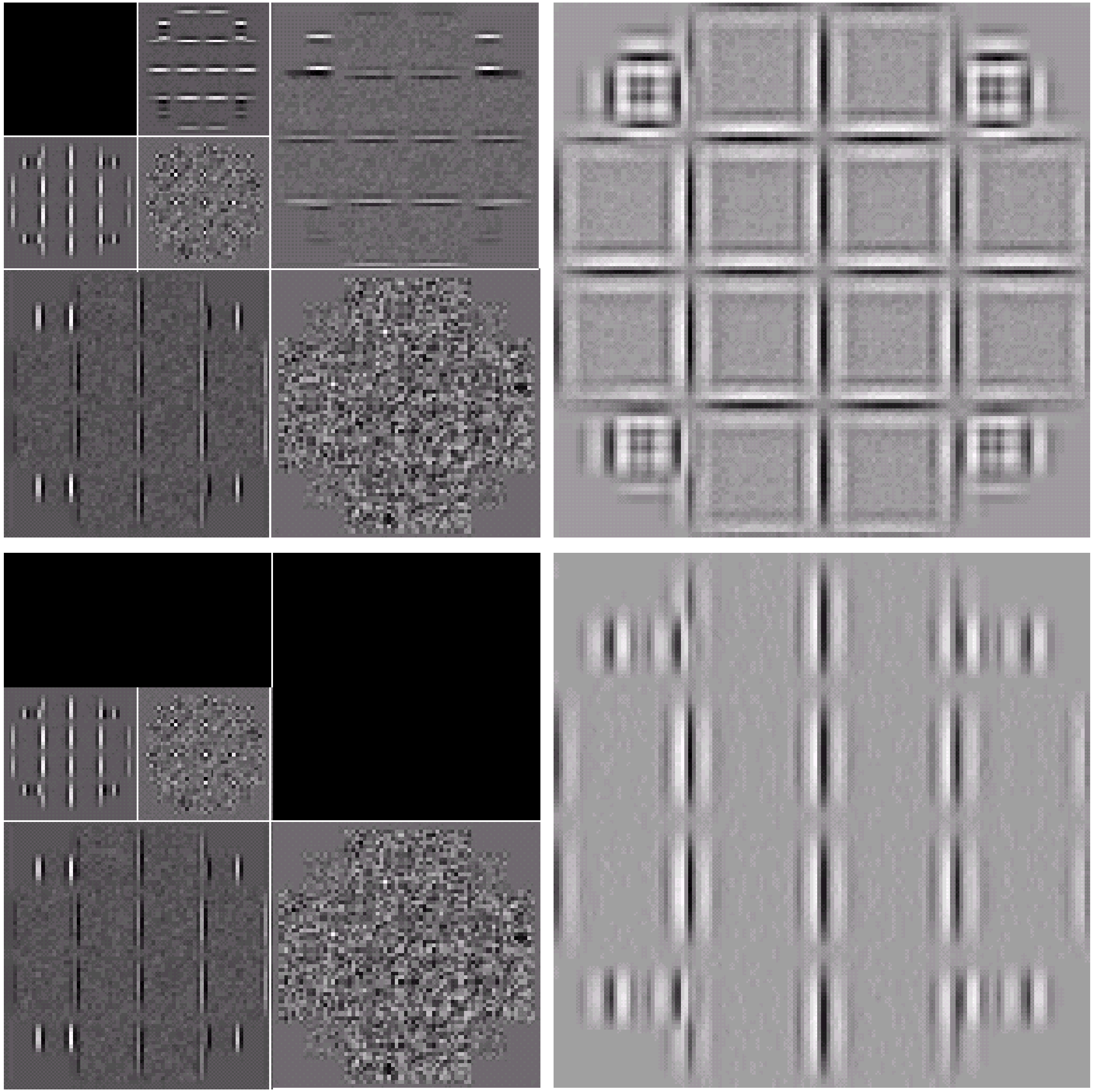


Fig. 7.24 (Con't)



a b
c d

FIGURE 7.25 Modifying a DWT for edge detection: (a) and (c) two-scale decompositions with selected coefficients deleted; (b) and (d) the corresponding reconstructions.



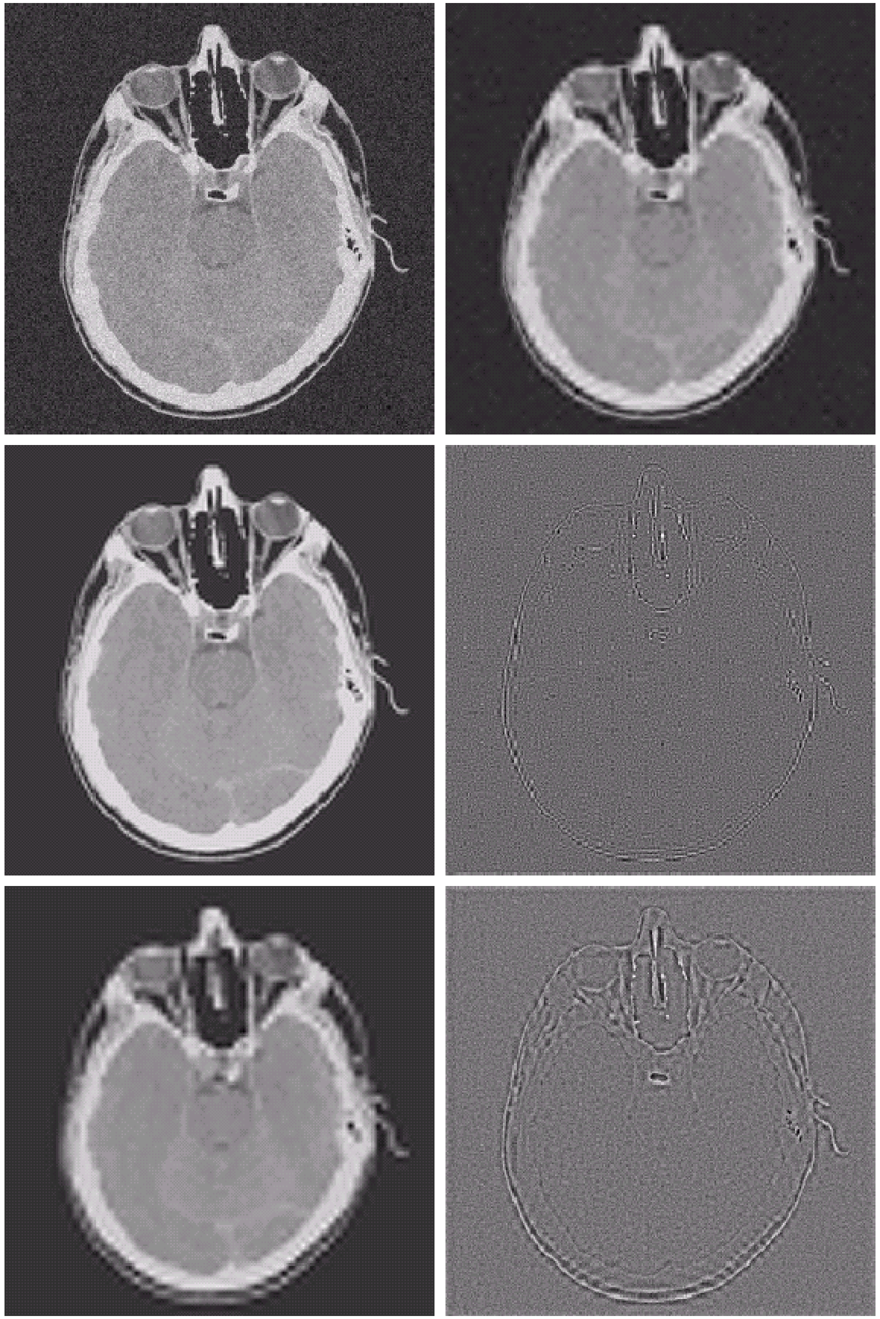


FIGURE 7.26 Modifying a DWT for noise removal: (a) a noisy MRI of a human head; (b), (c) and (e) various reconstructions after thresholding the detail coefficients; (d) and (f) the information removed during the reconstruction of (c) and (e). (Original image courtesy Vanderbuilt University Medical Center.)

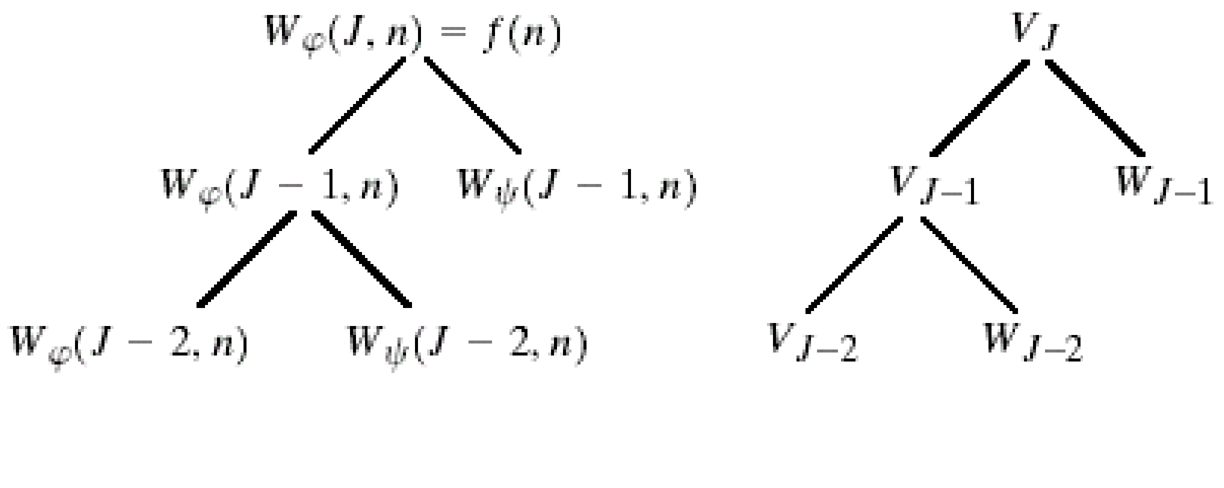


FIGURE 7.27 A coefficient (a) and analysis (b) tree for the two-scale FWT analysis bank of Fig. 7.16.

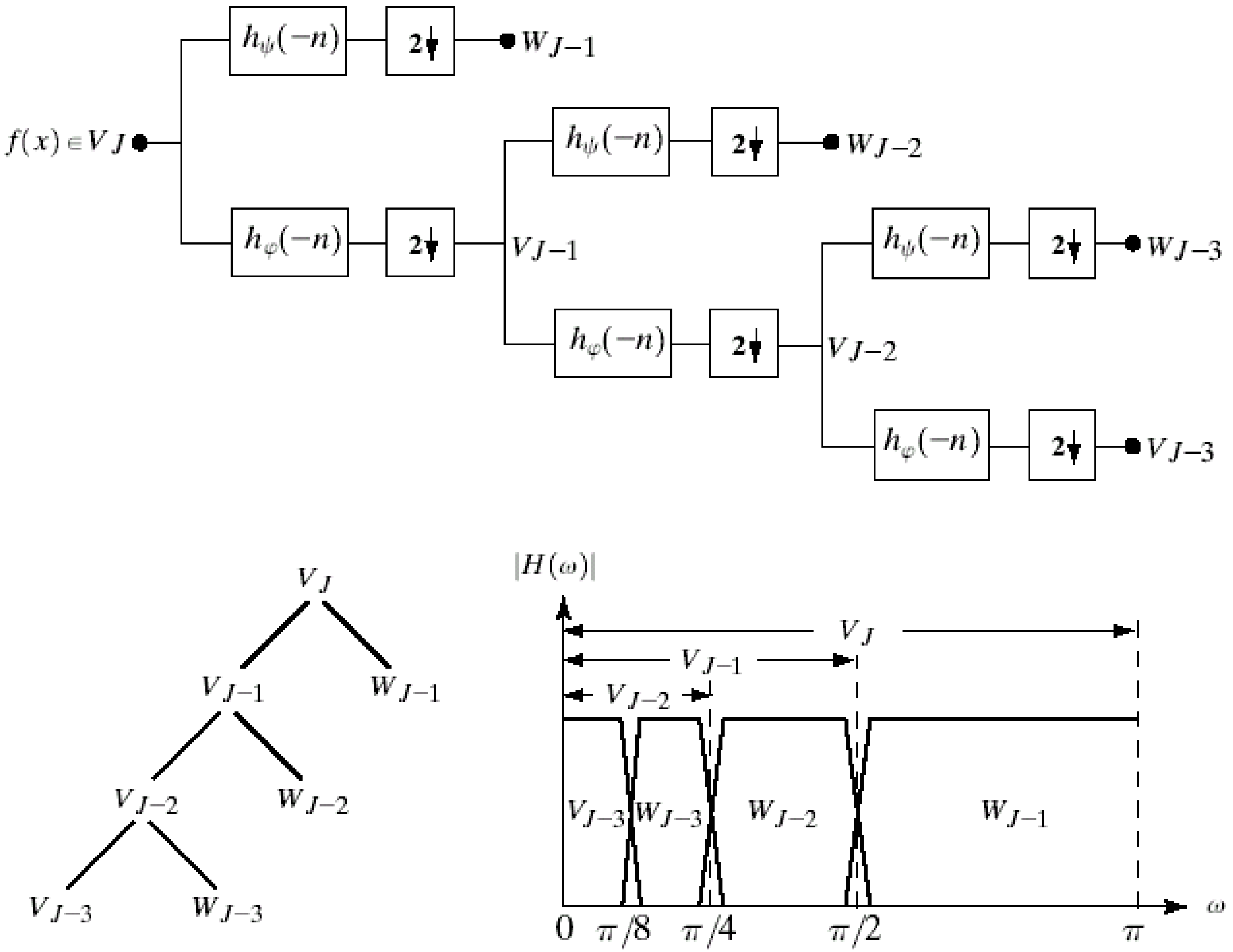


FIGURE 7.28 A three-scale FWT filter bank: (a) block diagram; (b) decomposition space tree; and (c) spectrum splitting characteristics.

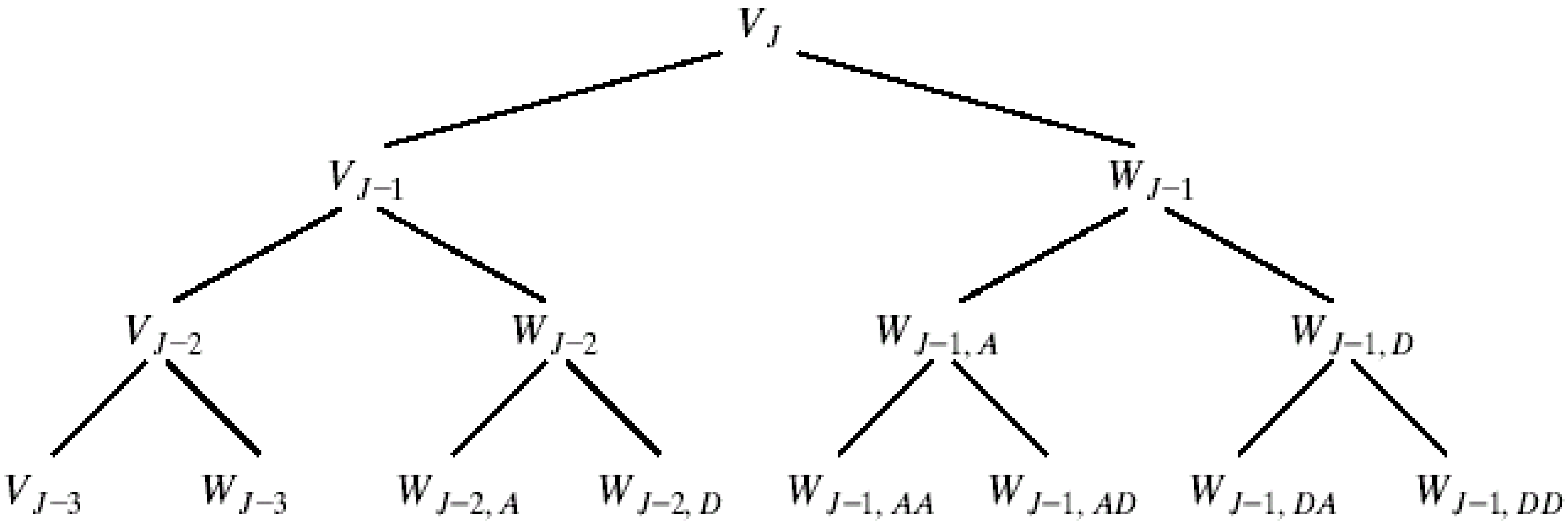
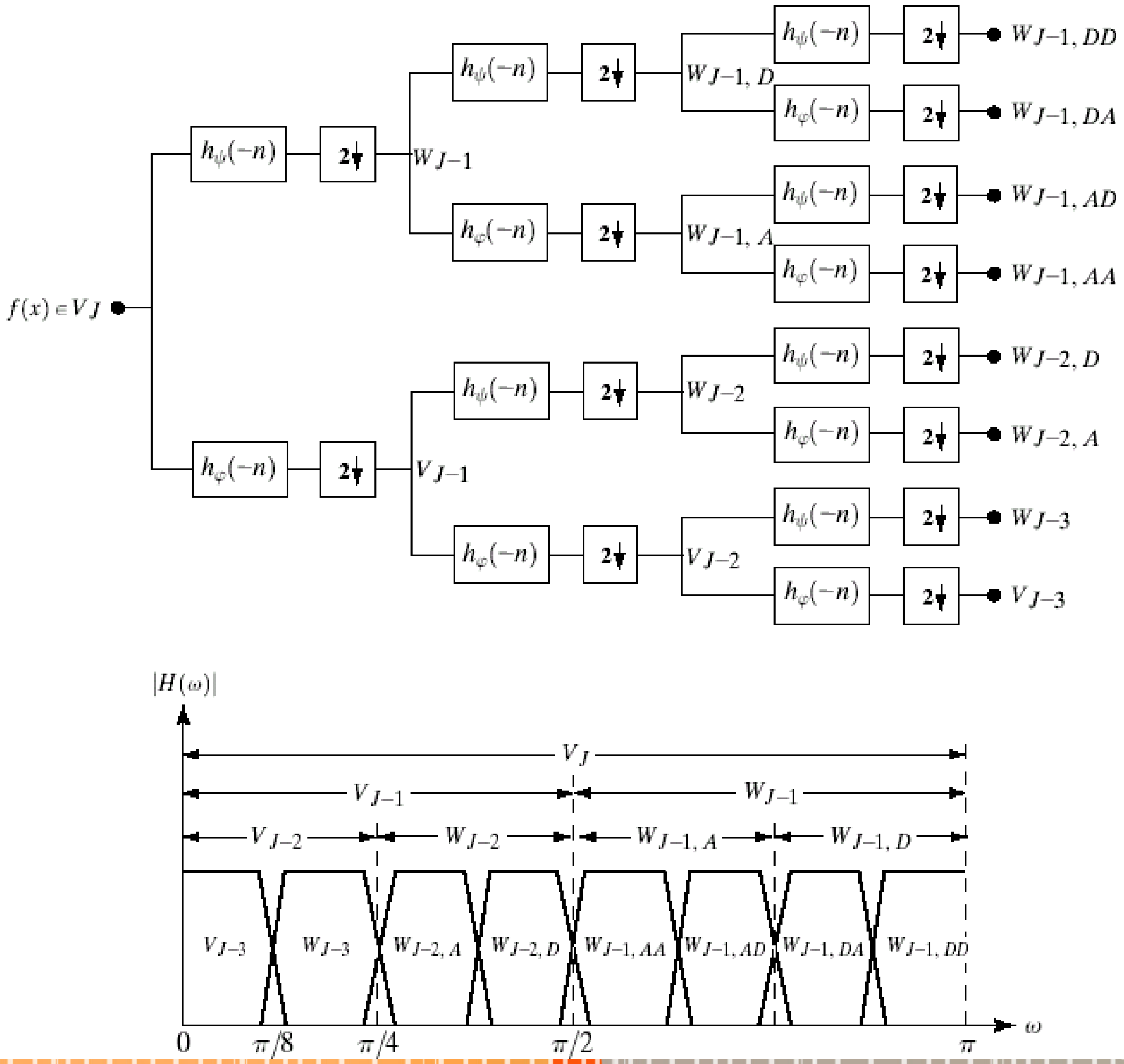


FIGURE 7.29 A three-scale wavelet packet analysis tree.



a
b

FIGURE 7.30 The (a) filter bank and (b) spectrum splitting characteristics of a three-scale full wavelet packet analysis tree.

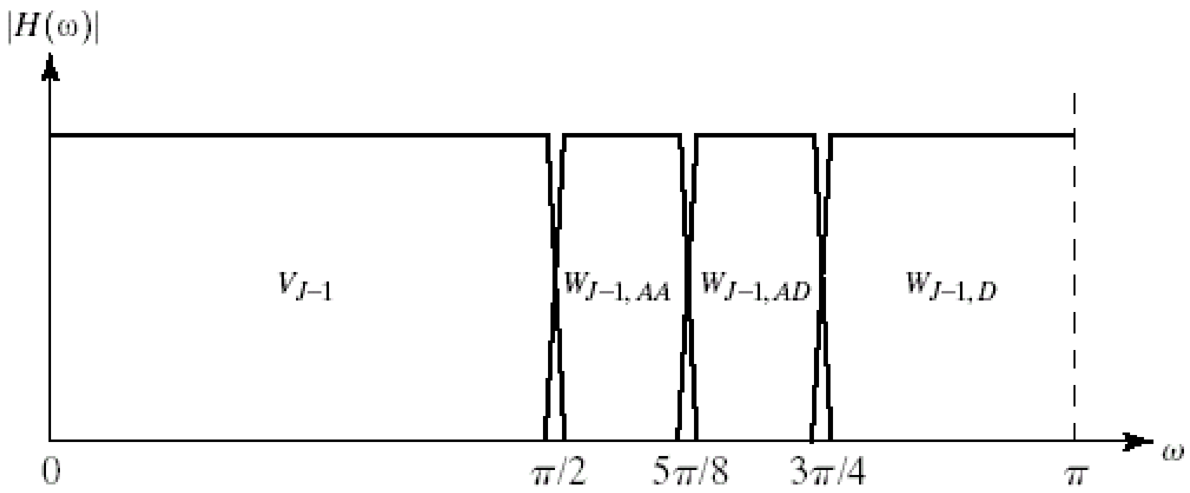


FIGURE 7.31 The spectrum of the decomposition in Eq. (7.6-5).

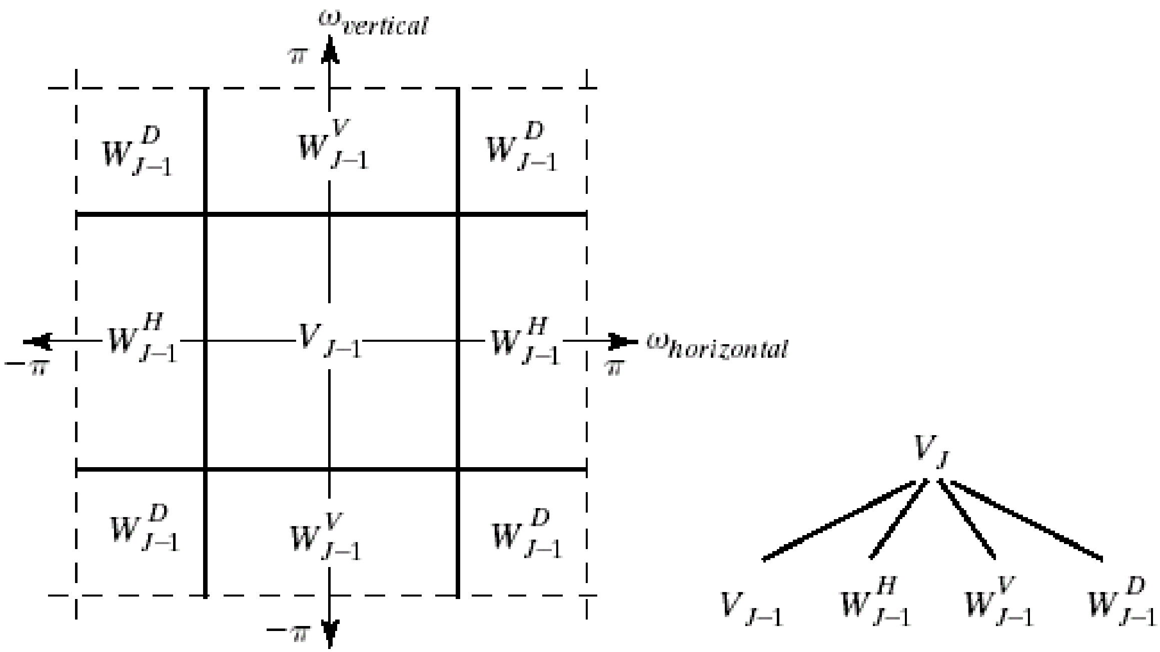


FIGURE 7.32 The first decomposition of a two-dimensional FWT: (a) the spectrum and (b) the subspace analysis tree.

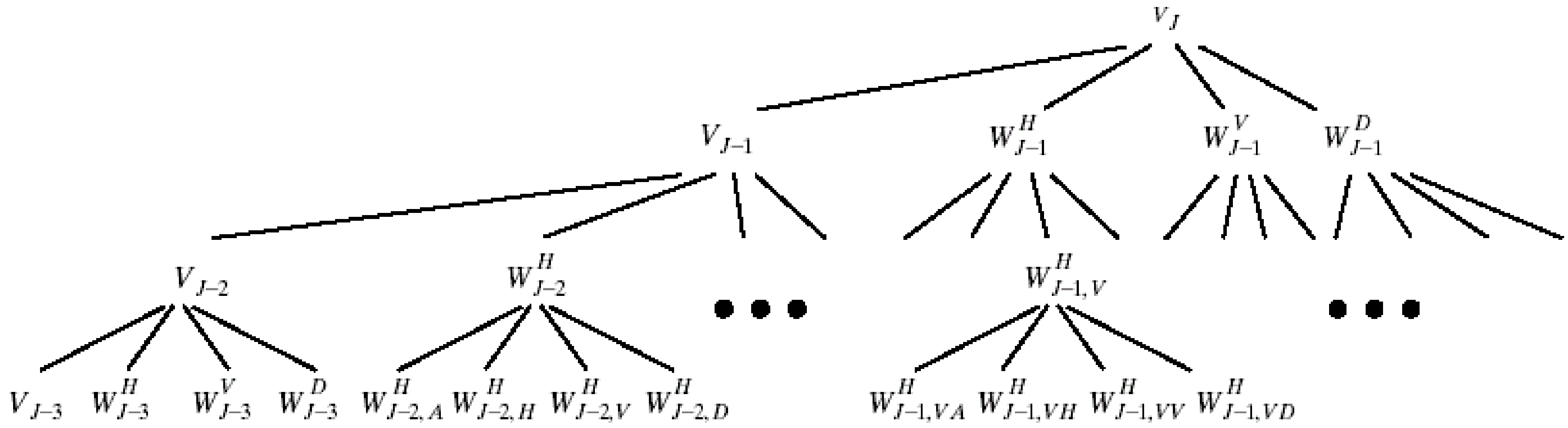
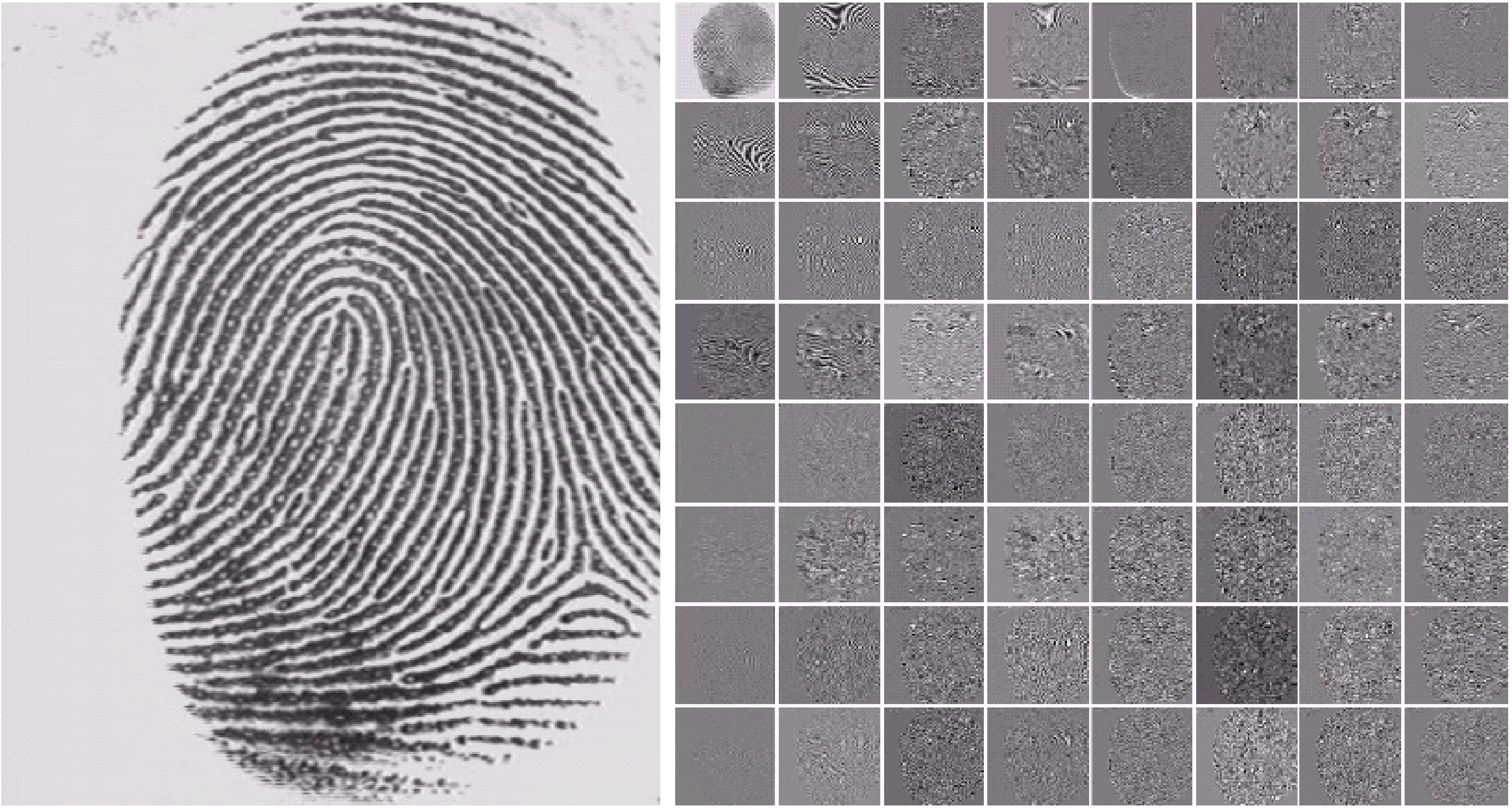


FIGURE 7.33 A three-scale, full wavelet packet decomposition tree. Only a portion of the tree is provided.



a b

FIGURE 7.34 (a) A scanned fingerprint and (b) its three-scale, full wavelet packet decomposition. (Original image courtesy of the National Institute of Standards and Technology.)

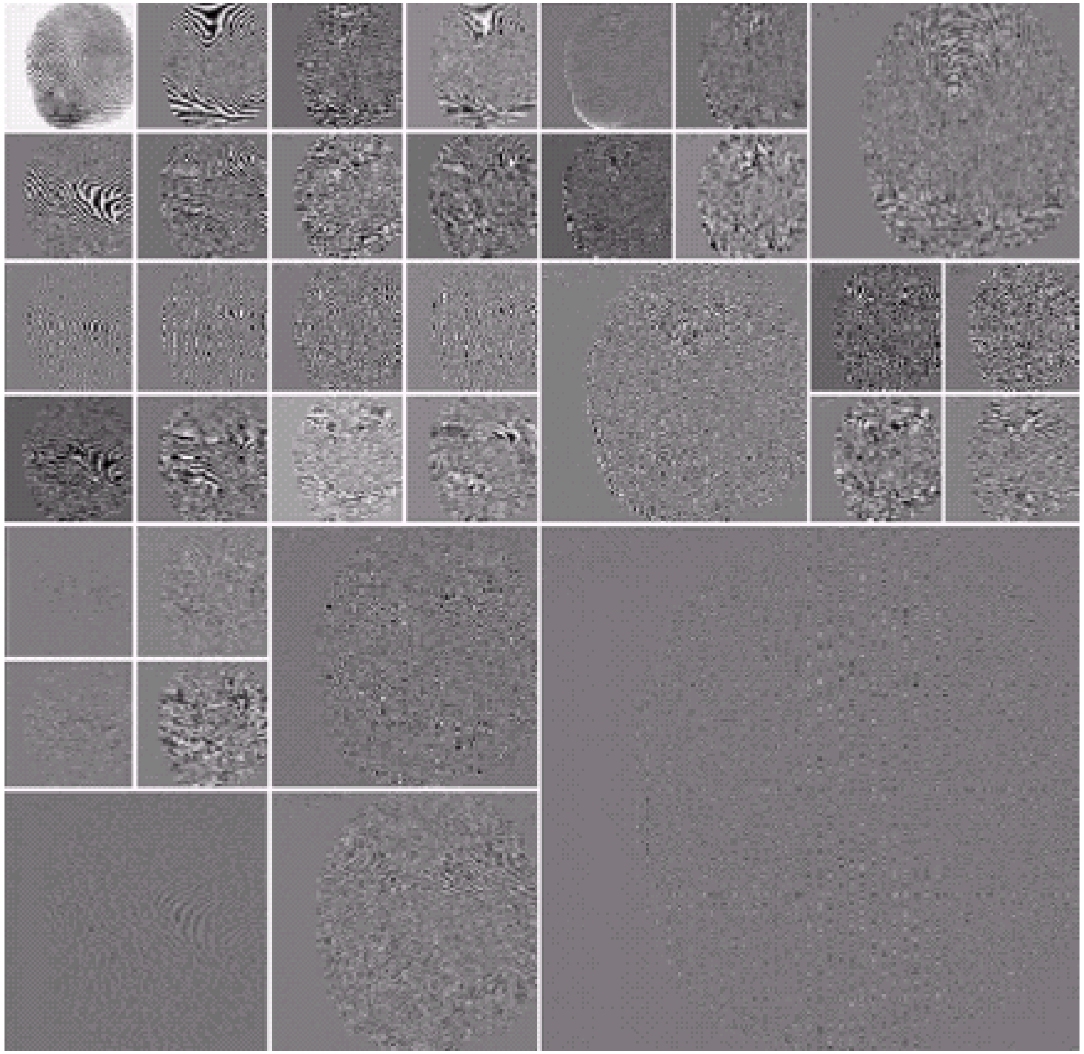


FIGURE 7.35 An optimal wavelet packet decomposition for the fingerprint of Fig. 7.34(a).

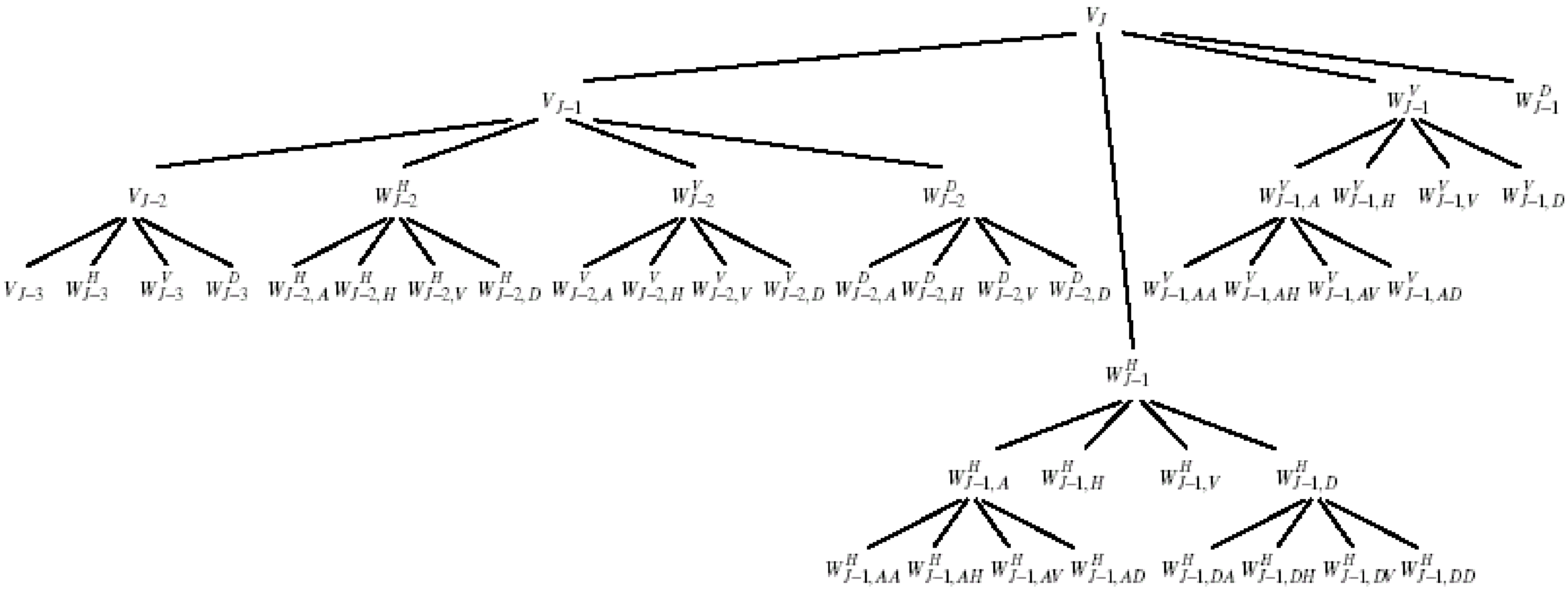
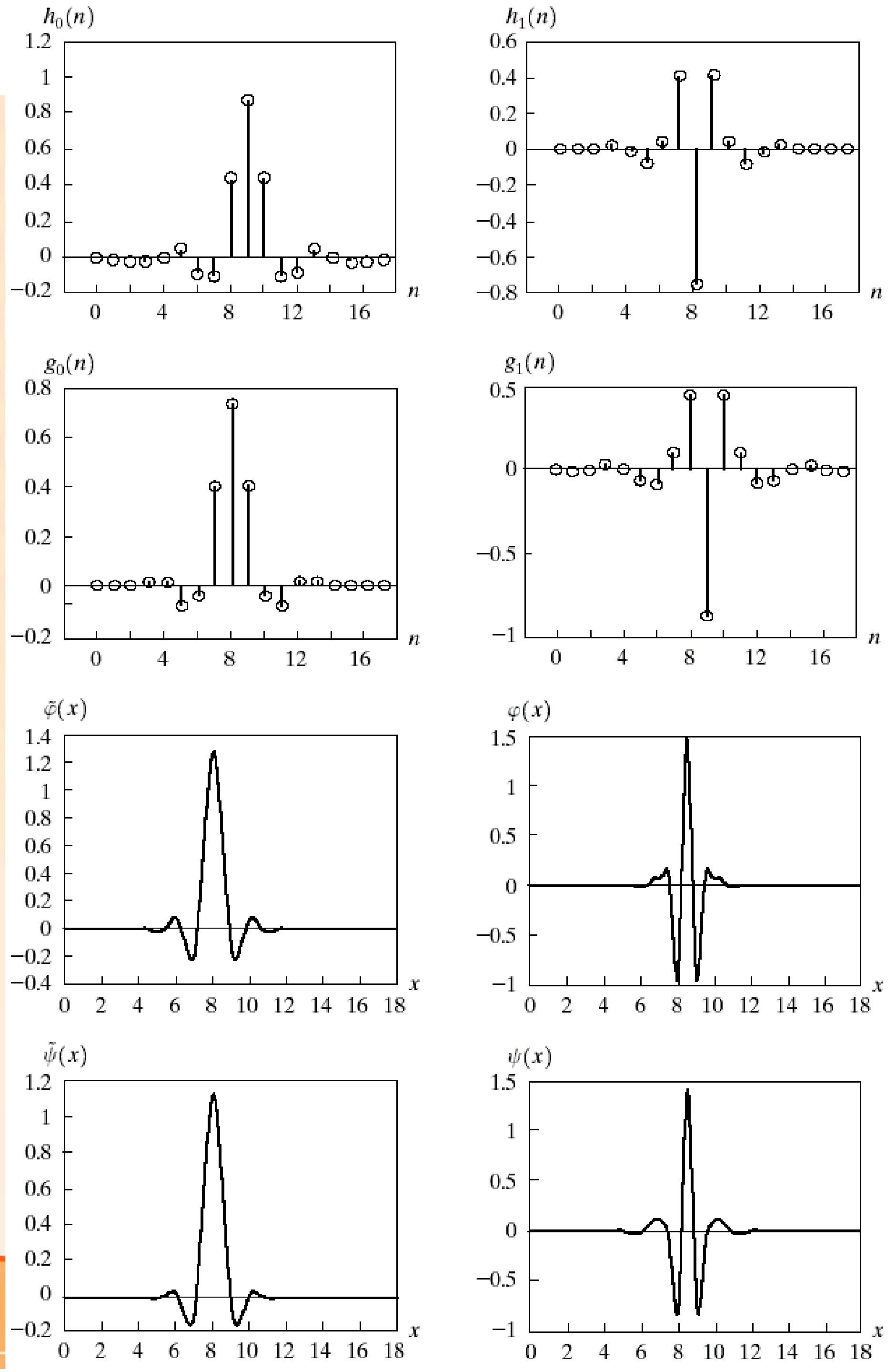


FIGURE 7.36 The optimal wavelet packet analysis tree for the decomposition in Fig. 7.35.



a b
c d
e f
g h

FIGURE 7.37 A member of the Cohen-Daubechies-Feauveau biorthogonal wavelet family: (a) and (b) decomposition filter coefficients; (c) and (d) reconstruction filter coefficients; (e)–(h) dual wavelet and scaling functions.