

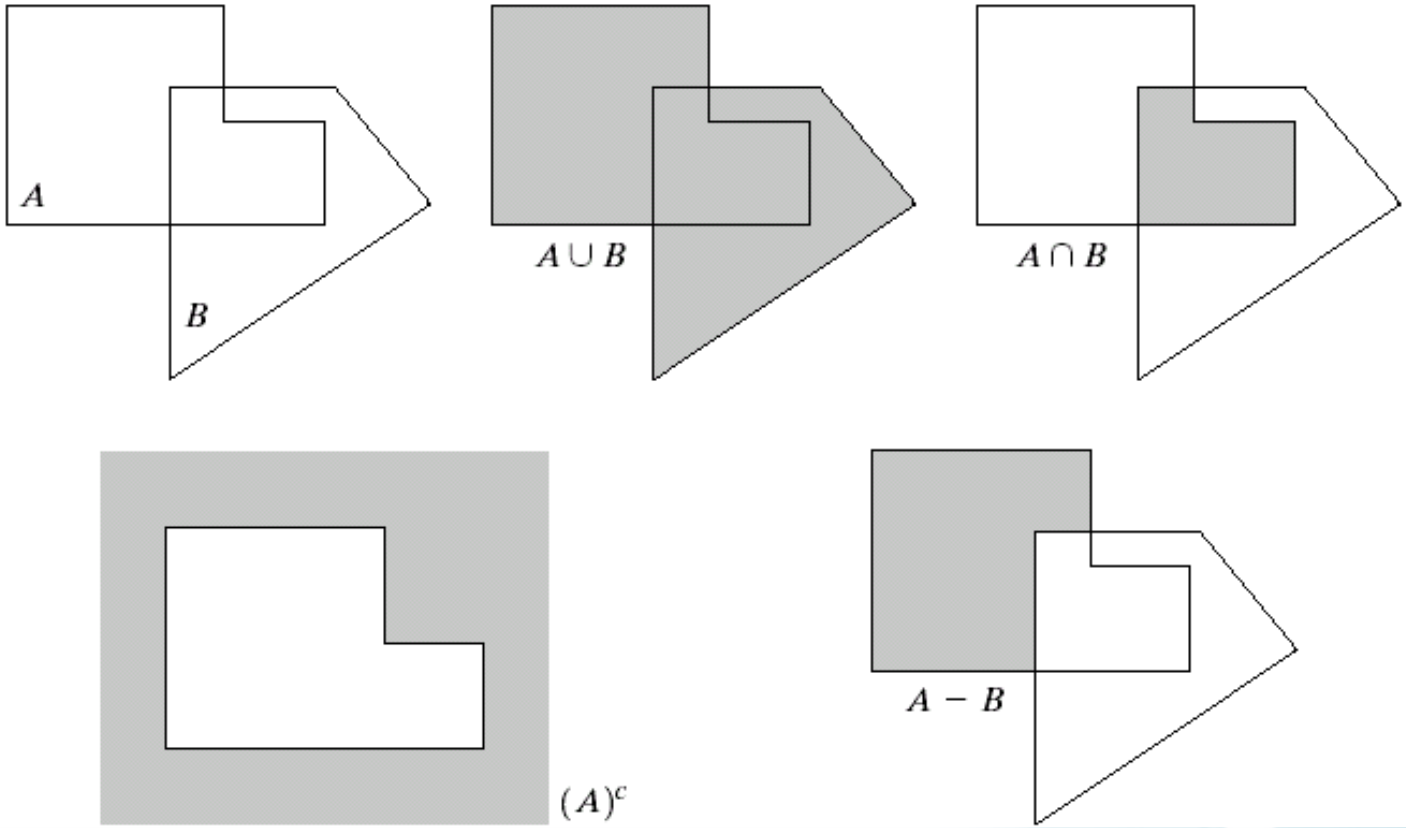

Chapter 9
Morphological Image Processing

IVPL

Preview

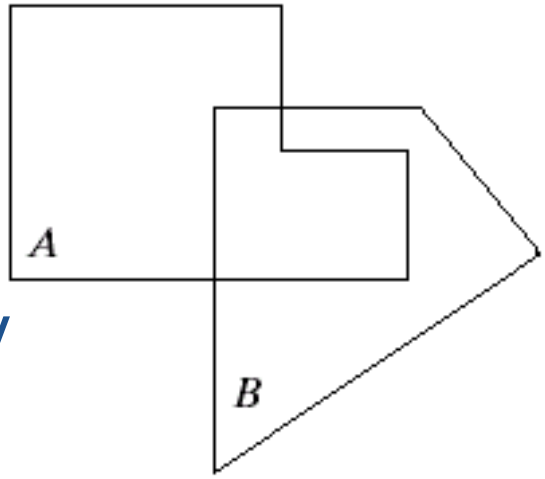
- Morphology 型態學
  - About the form and structure of animals and plants
- Mathematical morphology
  - Using set theory
  - Extract image component
  - Representation and description of region shape
- Sets in math. Morphology represent objects in an image
  - Example
    - Binary image: the elements of a set is the coordinate (x,y) of the pixels, in Z^2
    - Gray-level image: the element of a set is the triple, (x, y, gray-value), in Z^3

Basic Concepts



a b c
d e

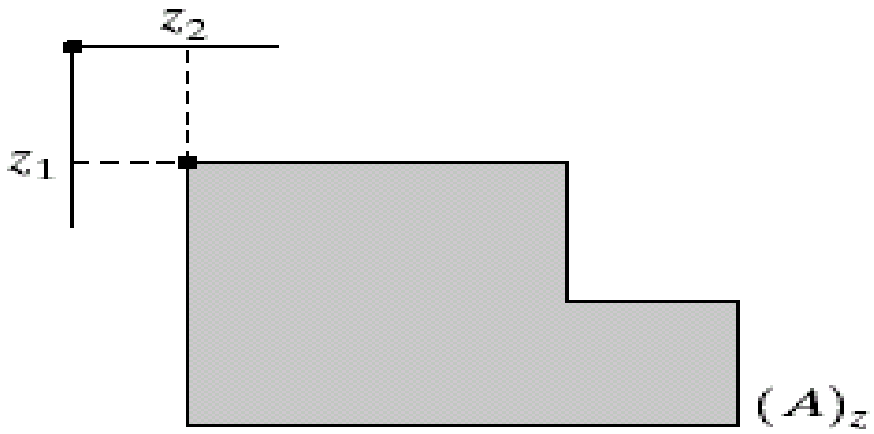
FIGURE 9.1
(a) Two sets A and B. (b) The union of A and B. (c) The intersection of A and B. (d) The complement of A. (e) The difference between A and B.



Special set operations for morphology

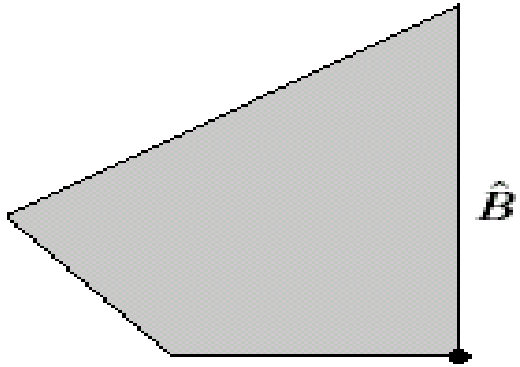
translation

$$(A)_z = \{c \mid c = a + z, \text{for } a \in A\}$$



reflection

$$\hat{B} = \{w \mid w = -b, \text{ for } b \in B\}$$



Logic Operations

TABLE 9.1
The three basic logical operations.

p	q	p AND q (also $p \cdot q$)	p OR q (also $p + q$)	NOT (p) (also $\bar{p}$)
0	0	0	0	1
0	1	0	1	1
1	0	0	1	0
1	1	1	1	0

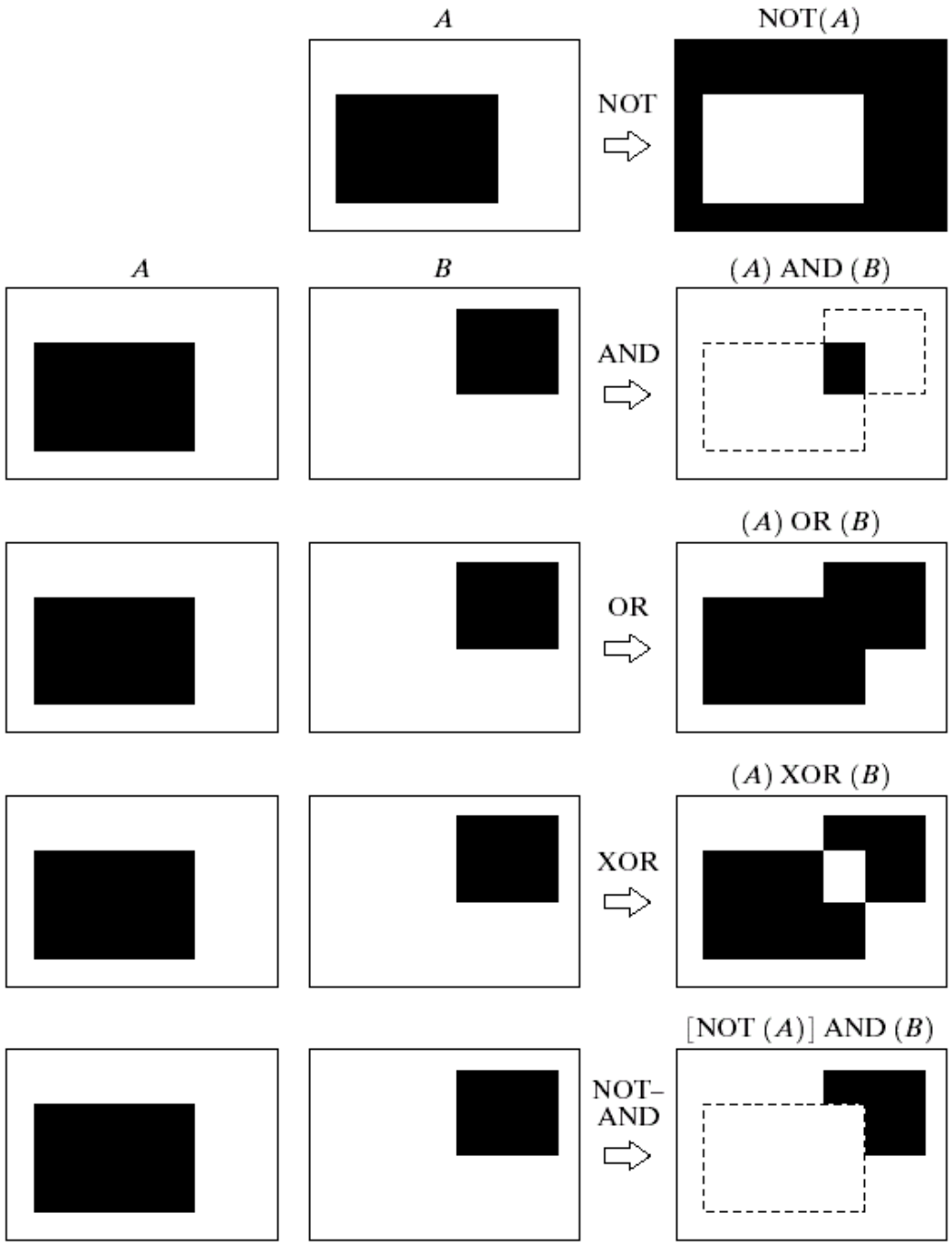
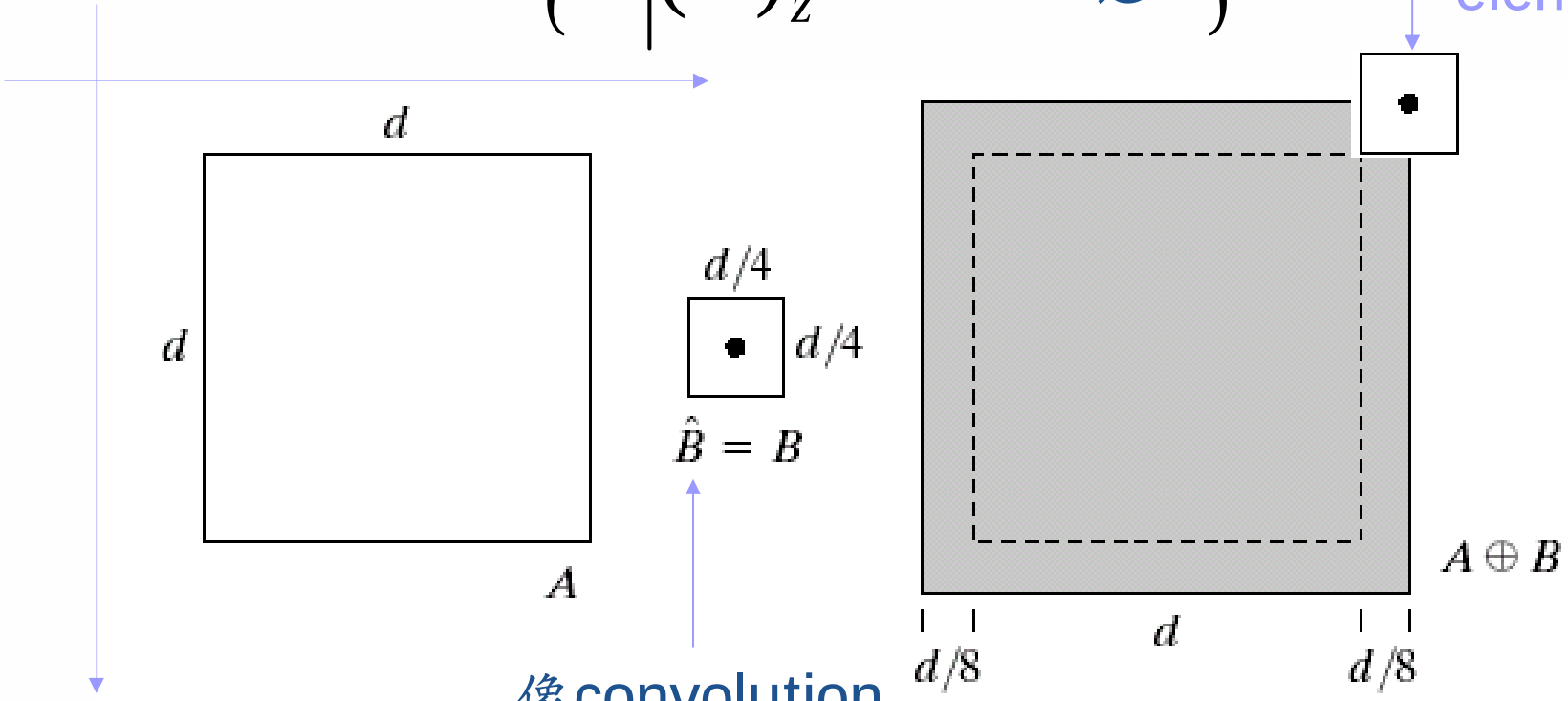


FIGURE 9.3 Some logic operations between binary images. Black represents binary 1s and white binary 0s in this example.

Dilation

$$A \oplus B = \left\{ z \middle| (\hat{B})_z \cap A \neq \varnothing \right\}$$

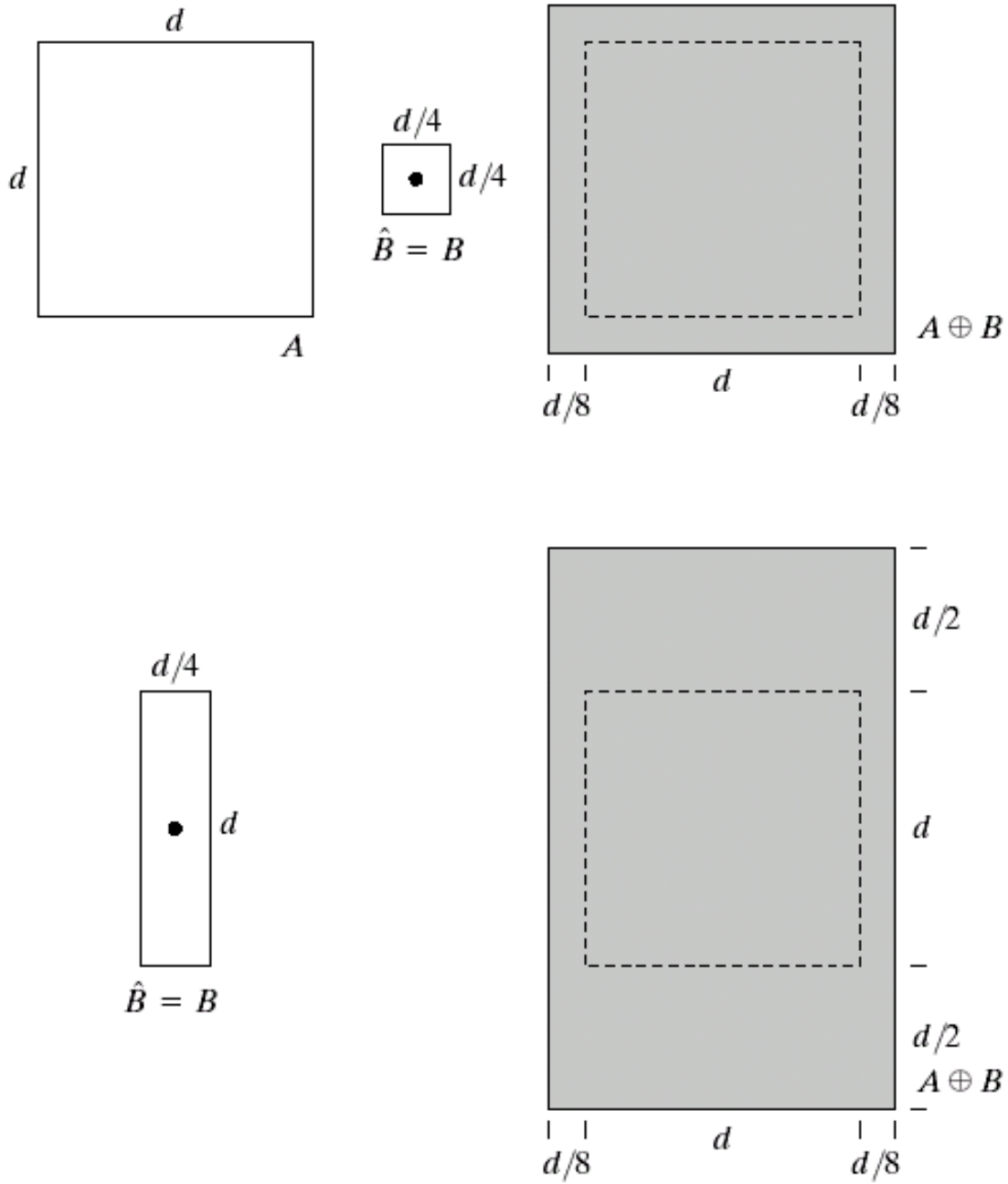
B:structuring element

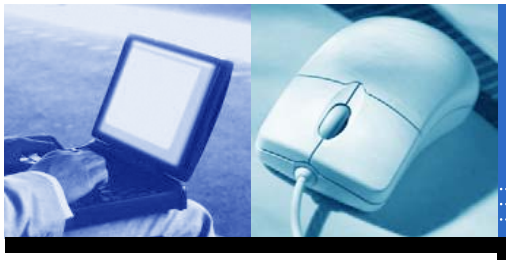


像convolution中filter翻轉

a b c
d e

FIGURE 9.4
(a) Set A.
(b) Square structuring element (dot is the center).
(c) Dilation of A by B, shown shaded.
(d) Elongated structuring element.
(e) Dilation of A using this element.



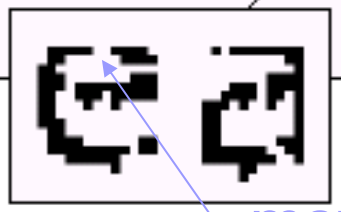


Application of dilation: bridging gaps in images

0	1	0
1	1	1
0	1	0

Structuring element

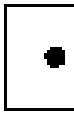
Historically, certain computer programs were written using only two digits rather than four to define the applicable year. Accordingly, the company's software may recognize a date using "00" as 1900 rather than the year 2000.



max. gap=2 pixels

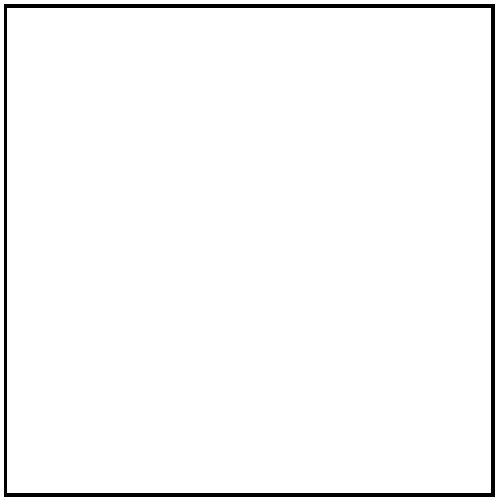
Historically, certain computer programs were written using only two digits rather than four to define the applicable year. Accordingly, the company's software may recognize a date using "00" as 1900 rather than the year 2000.

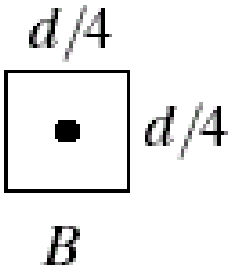
Erosion 腐蝕

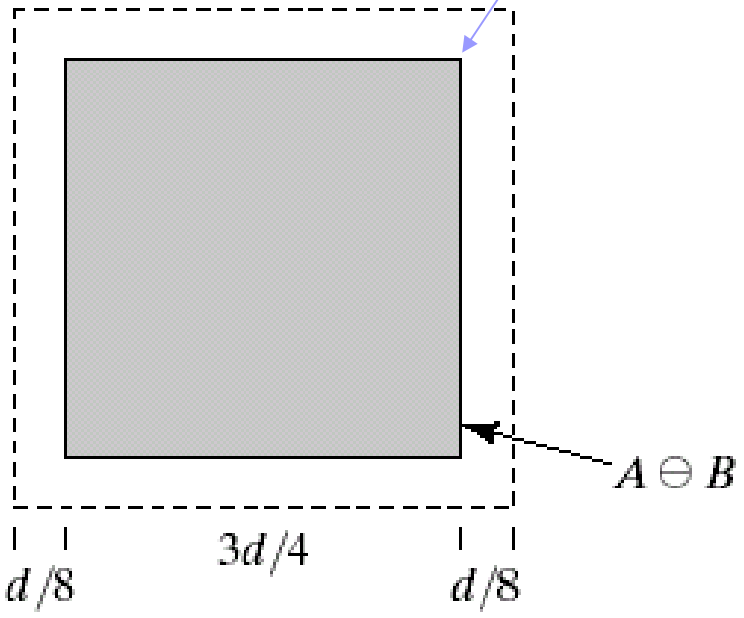


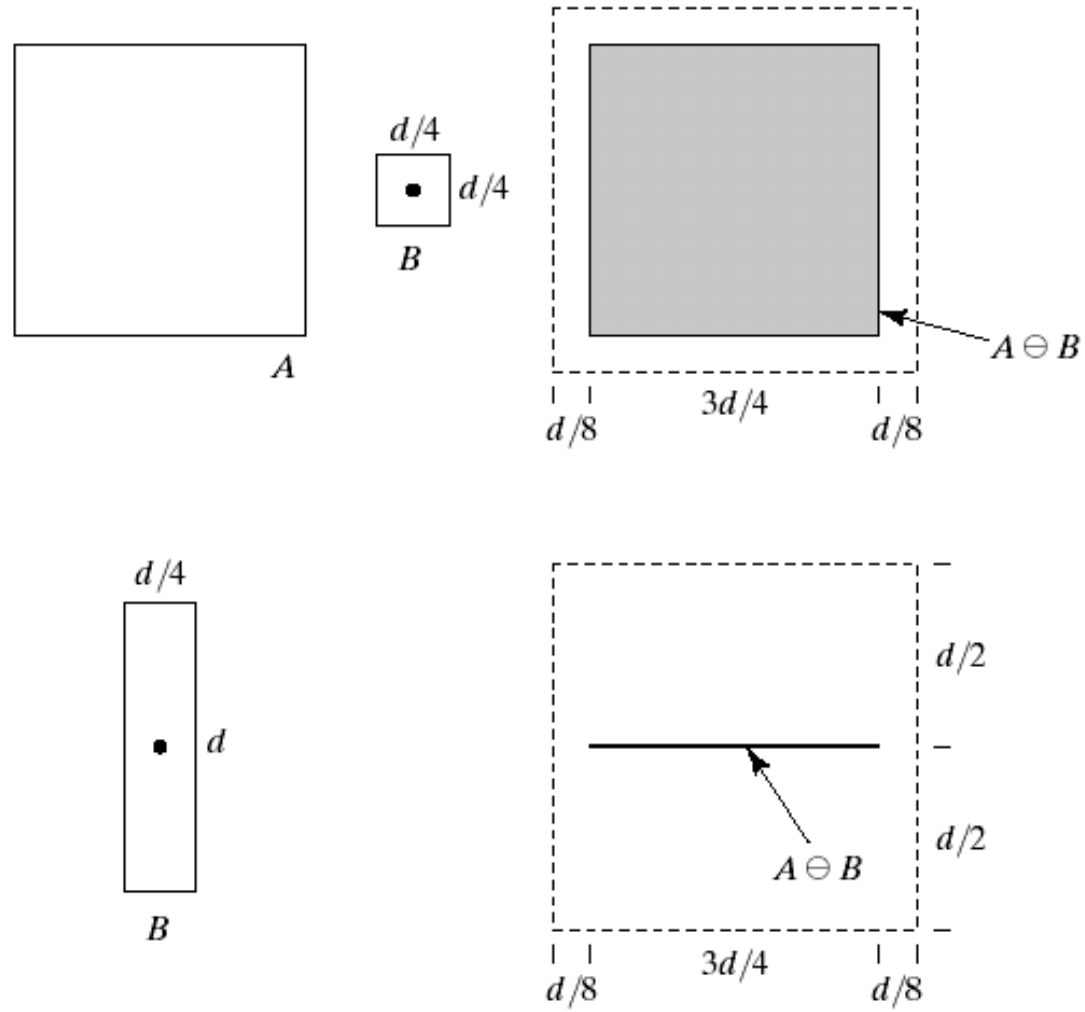
$$A \ominus B = \left\{ z \middle| (B)_z \subseteq A \right\}$$

z: displacement









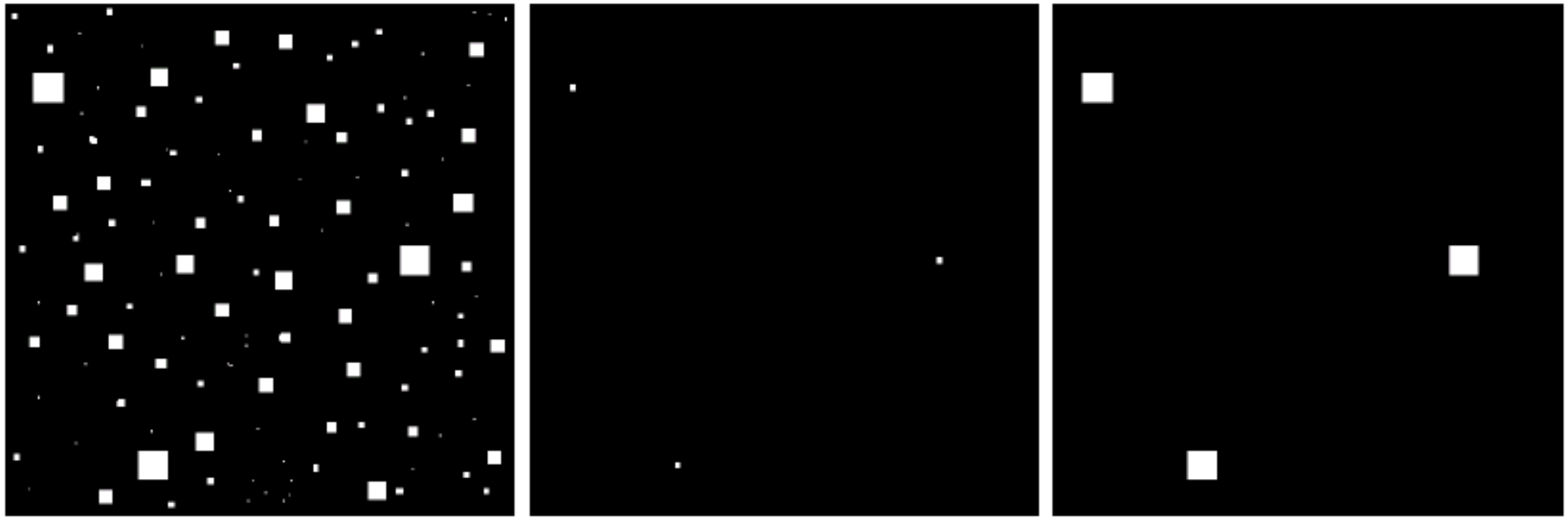
a b c
d e

FIGURE 9.6 (a) Set A. (b) Square structuring element. (c) Erosion of A by B, shown shaded. (d) Elongated structuring element. (e) Erosion of A using this element.

Application of erosion: eliminate irrelevant detail

Squares of size
1,3,5,7,9,15 pels

Erode with
13x13 square



original image → erosion → dilation

Dilation and erosion are duals

$$(A \ominus B)^c = \left\{ z \middle| (B)_z \subseteq A \right\}^c$$

$$= \left\{ z \middle| (B)_z \cap A^c = \varnothing \right\}^c$$

$$= \left\{ z \middle| (B)_z \cap A^c \neq \varnothing \right\}$$

$$= A^c \oplus \hat{B}$$

$$A \oplus B = \left\{ z \middle| (\hat{B})_z \cap A \neq \varnothing \right\}$$

9.3 Opening and Closing

- Opening – $\circ$

  $A \circ B = (A \ominus B) \oplus B.$

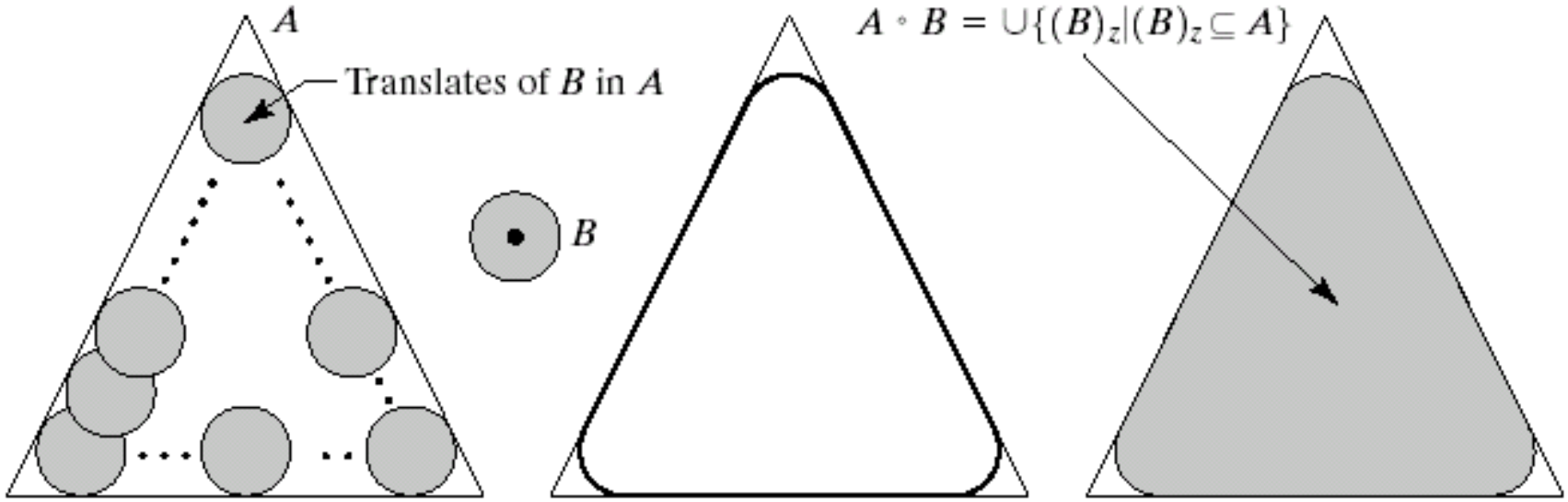
- Closing - $\bullet$

  $A \bullet B = (A \oplus B) \ominus B.$

- Duals

$$(A \bullet B)^C = (A^C \circ \hat{B}).$$

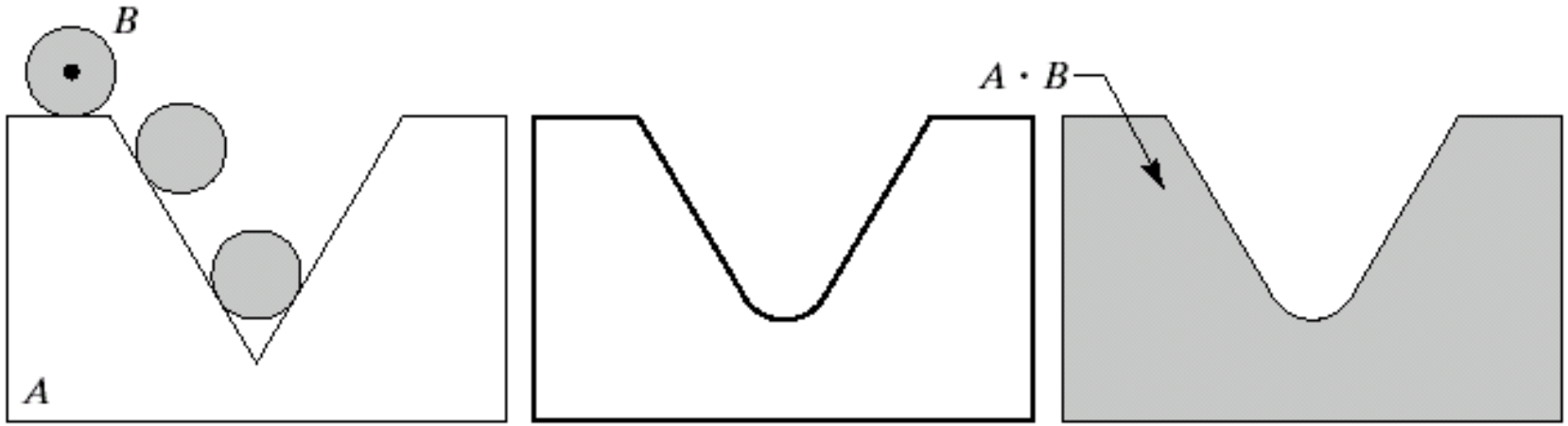
Opening



a b c d

FIGURE 9.8 (a) Structuring element B "rolling" along the inner boundary of A (the dot indicates the origin of B). (c) The heavy line is the outer boundary of the opening. (d) Complete opening (shaded).

Closing

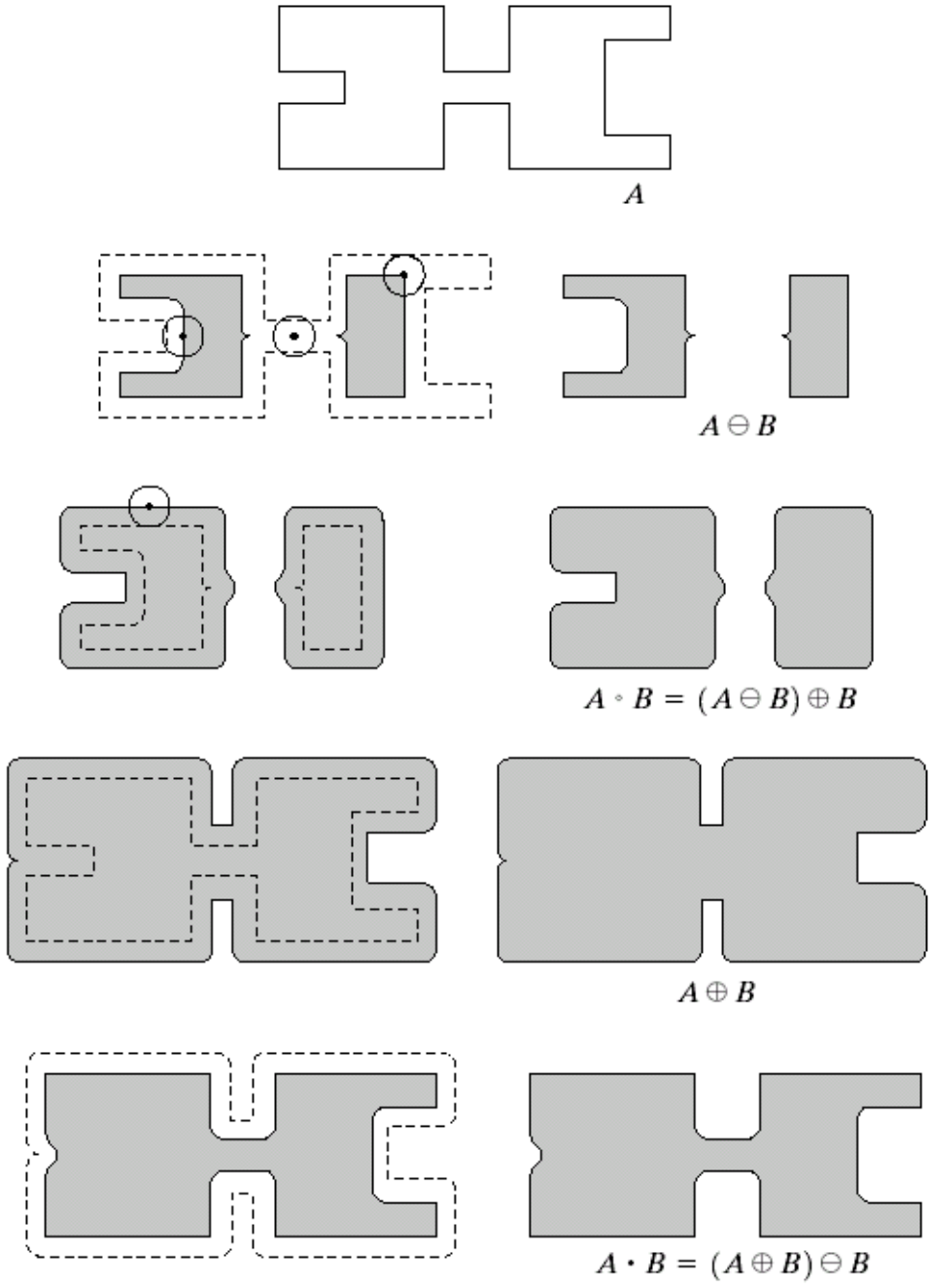


a b c

FIGURE 9.9 (a) Structuring element B "rolling" on the outer boundary of set A. (b) Heavy line is the outer boundary of the closing. (c) Complete closing (shaded).

a
b c
d e
f g
h i

FIGURE 9.10
Morphological opening and closing. The structuring element is the small circle shown in various positions in (b). The dark dot is the center of the structuring element.



Noisy image

Remove outer noise

A

$A \ominus B$

1	1	1
1	1	1
1	1	1

B

opening

$(A \ominus B) \oplus B = A \circ B$

$(A \circ B) \oplus B$

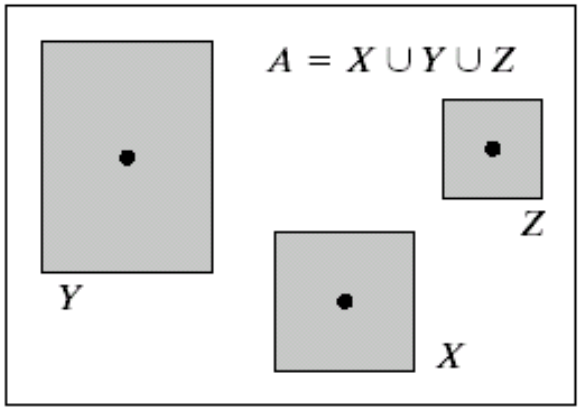
$[(A \circ B) \oplus B] \ominus B = (A \circ B) \cdot B$

closing

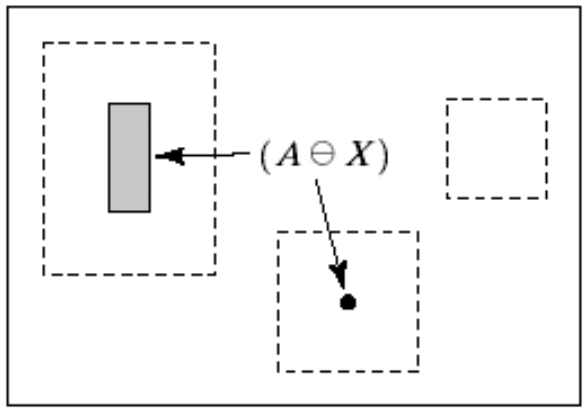
Remove inner noise

Hit-or-miss transformation

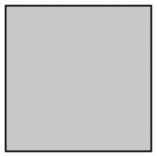
- Find the location of certain shape



erosion



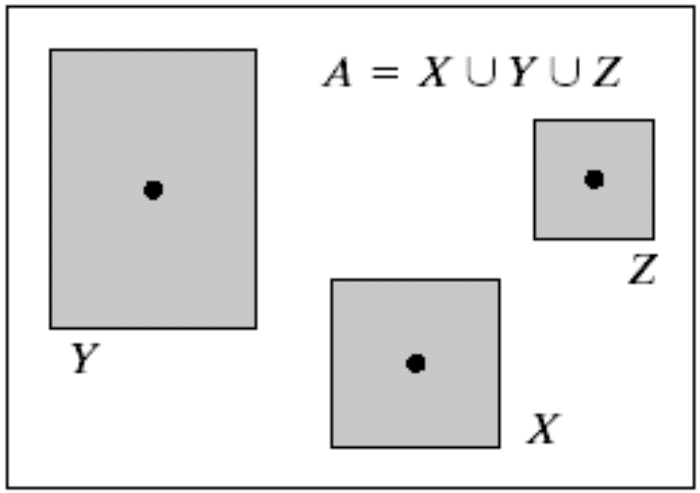
X

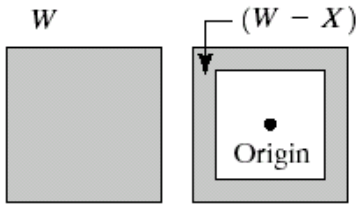


Find the set of pixels that contain shape W

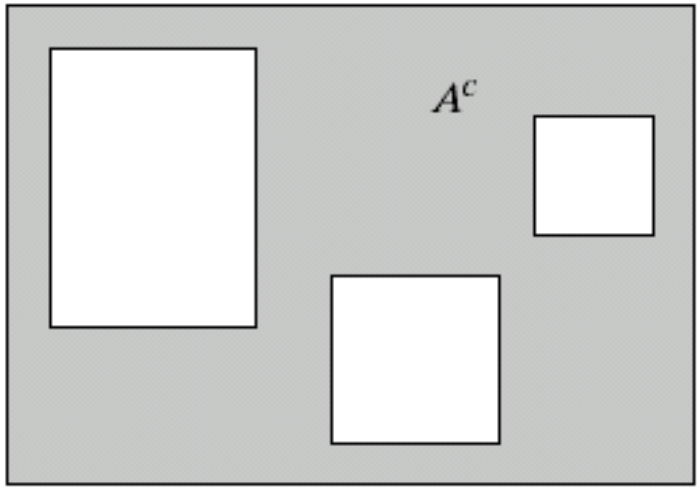
如何只找到相符形狀中心點？

Hit-or-miss transformation (cont.)

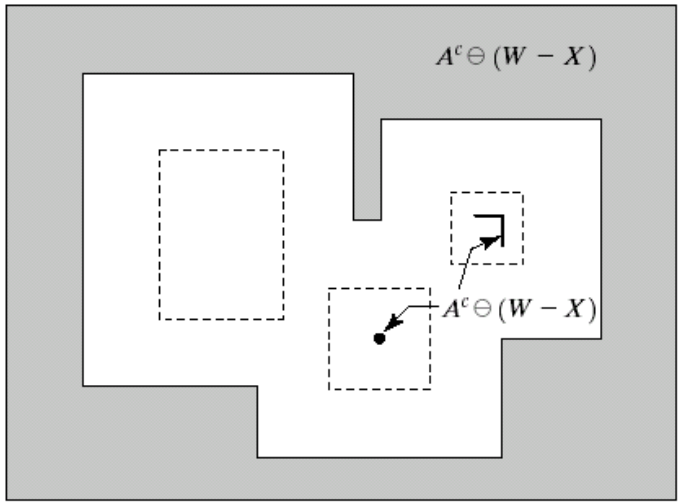




Detect object via background

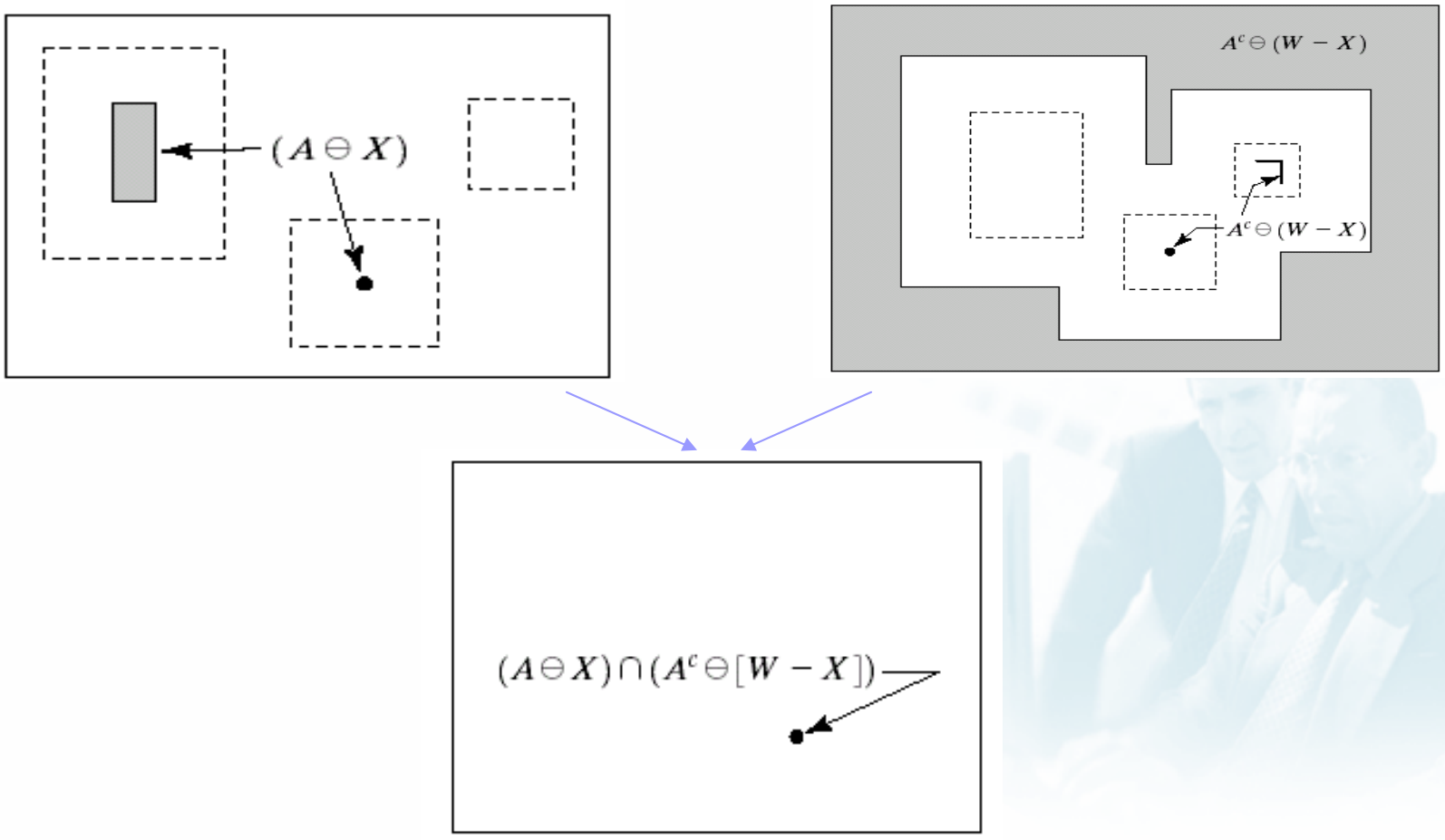


Erosion with (W-X)



Hit-or-miss transformation (cont.)

- Eliminate un-necessary parts



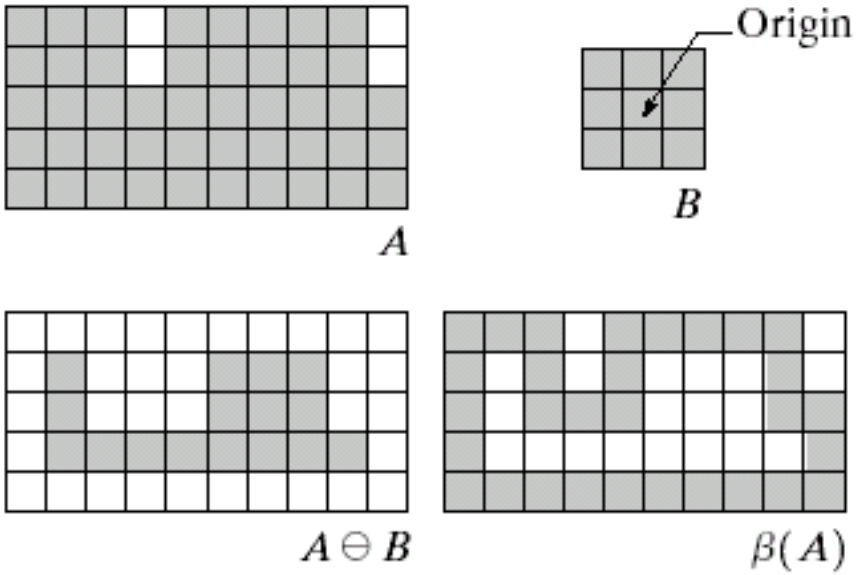
9.5 Morphological Algorithms

- Boundary Extraction

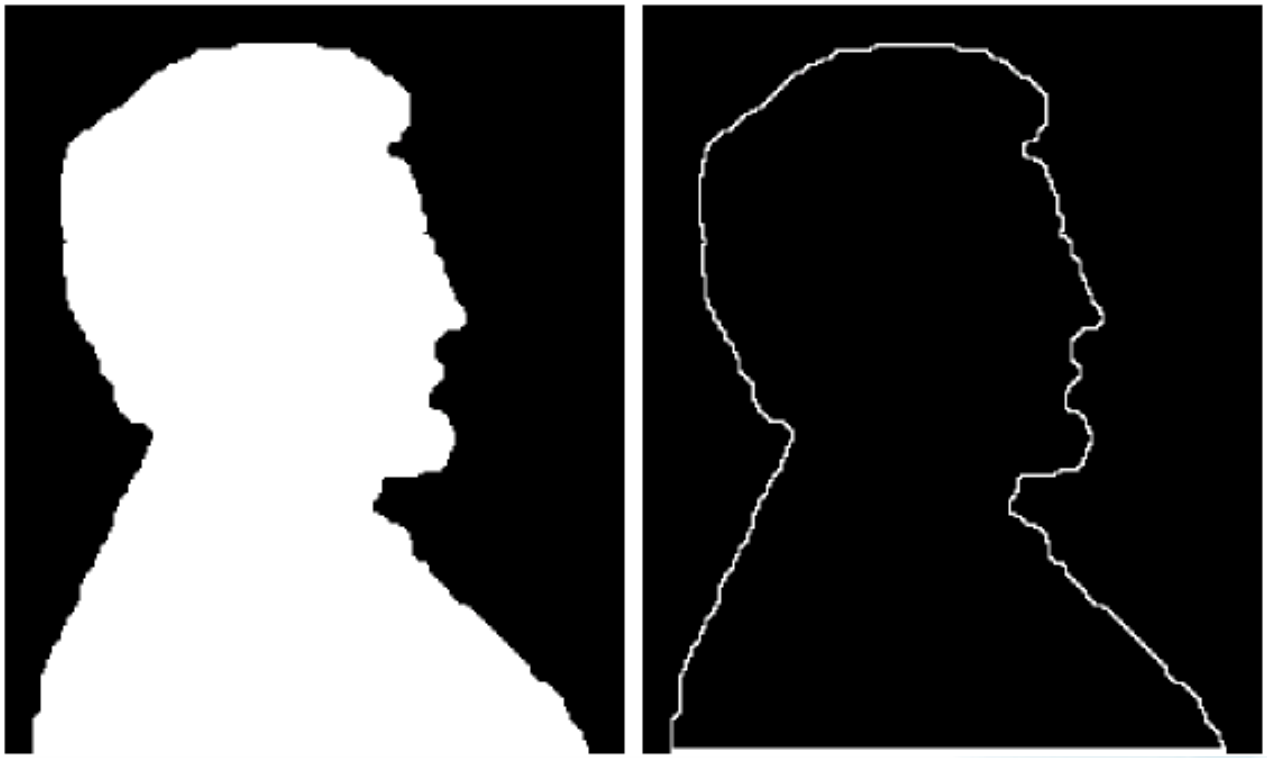
  ß (A) = A – (A Θ B)

a b
c d

FIGURE 9.13 (a) Set A. (b) Structuring element B. (c) A eroded by B. (d) Boundary, given by the set difference between A and its erosion.



Example 9.5

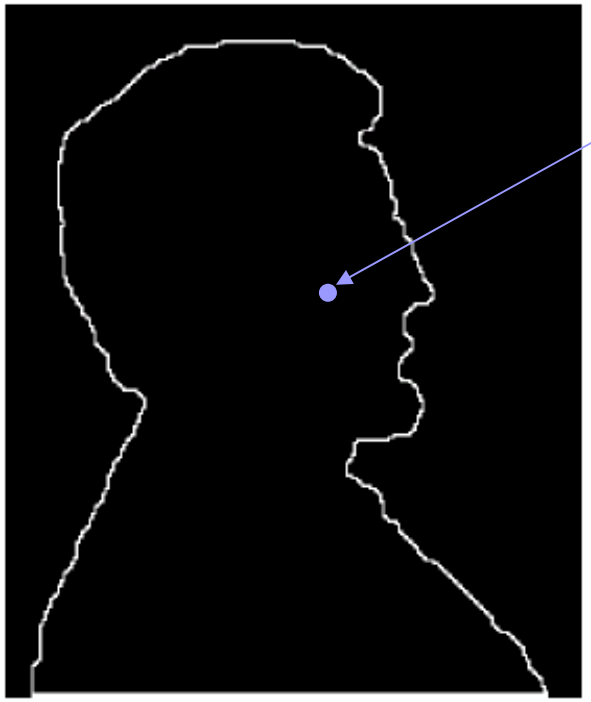


a b

FIGURE 9.14
(a) A simple binary image, with 1's represented in white. (b) Result of using Eq. (9.5-1) with the structuring element in Fig. 9.13(b).

Region filling

- Idea: place a point inside the region, then dilate that point iteratively



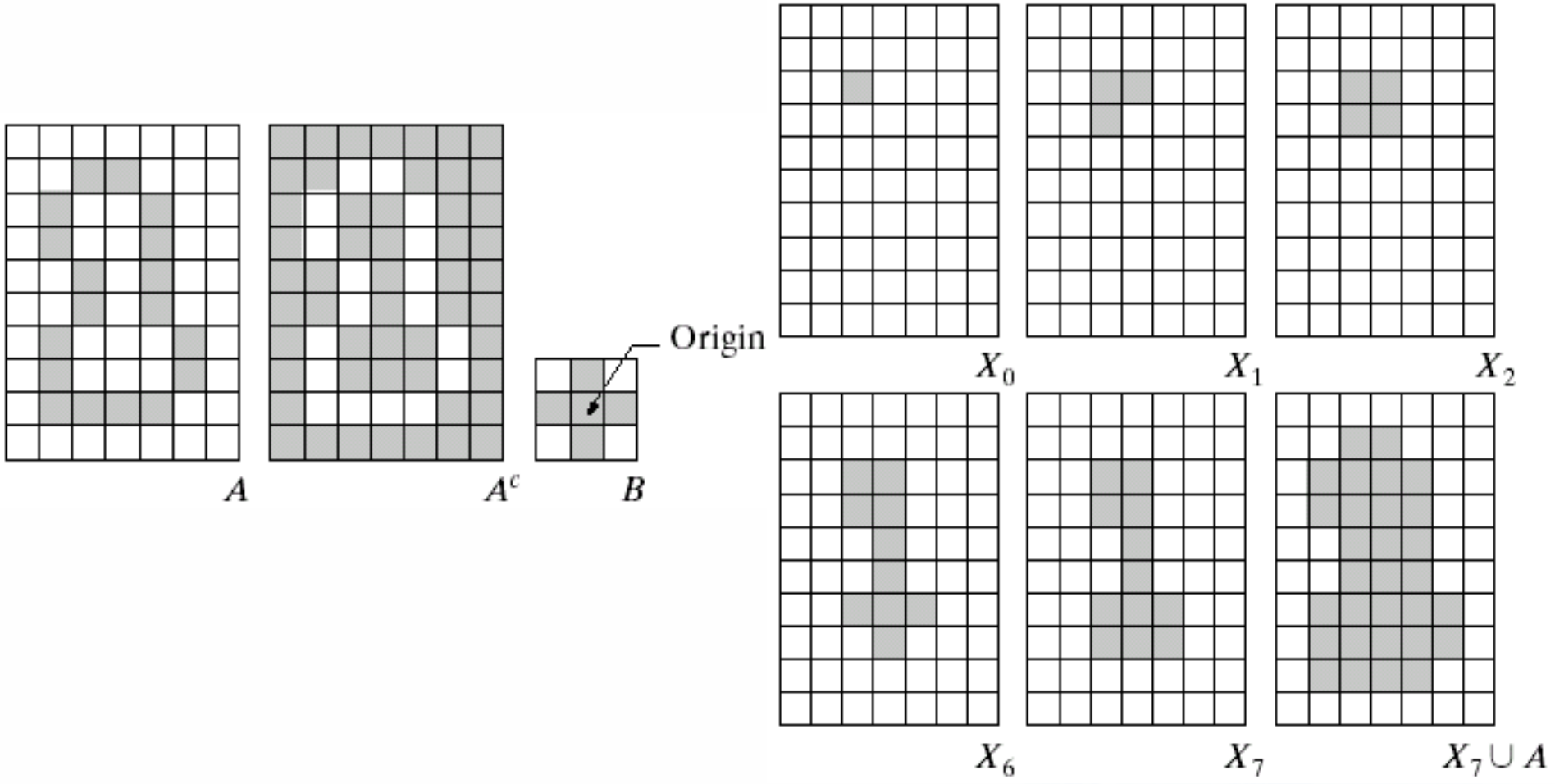
$$X_0 = p$$

$$X_k = (X_{k-1} \oplus B) \cap A^c, k = 1,2,3,...$$

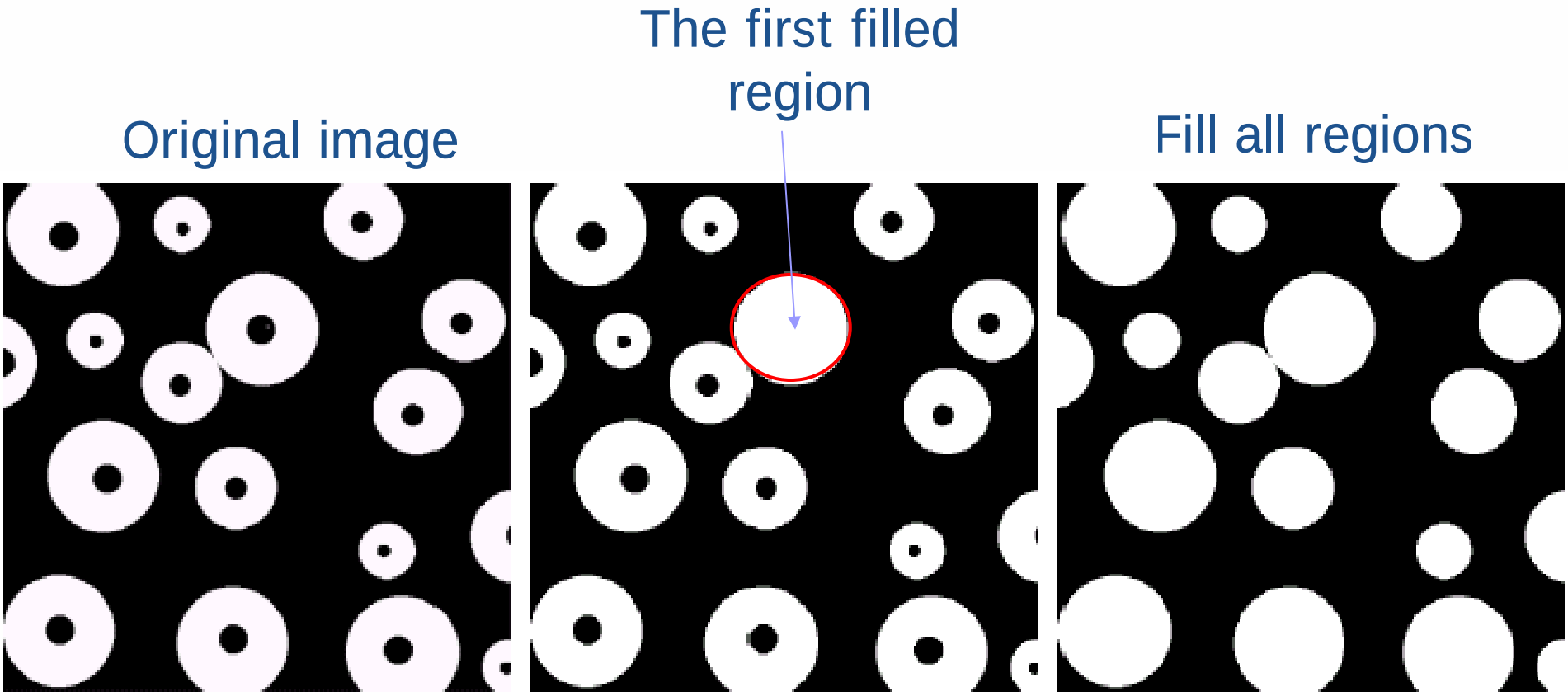
Until $X_k = X_{k-1}$

Bound the growth

Region filling (cont.)



Application: region filling



Extraction of connected components

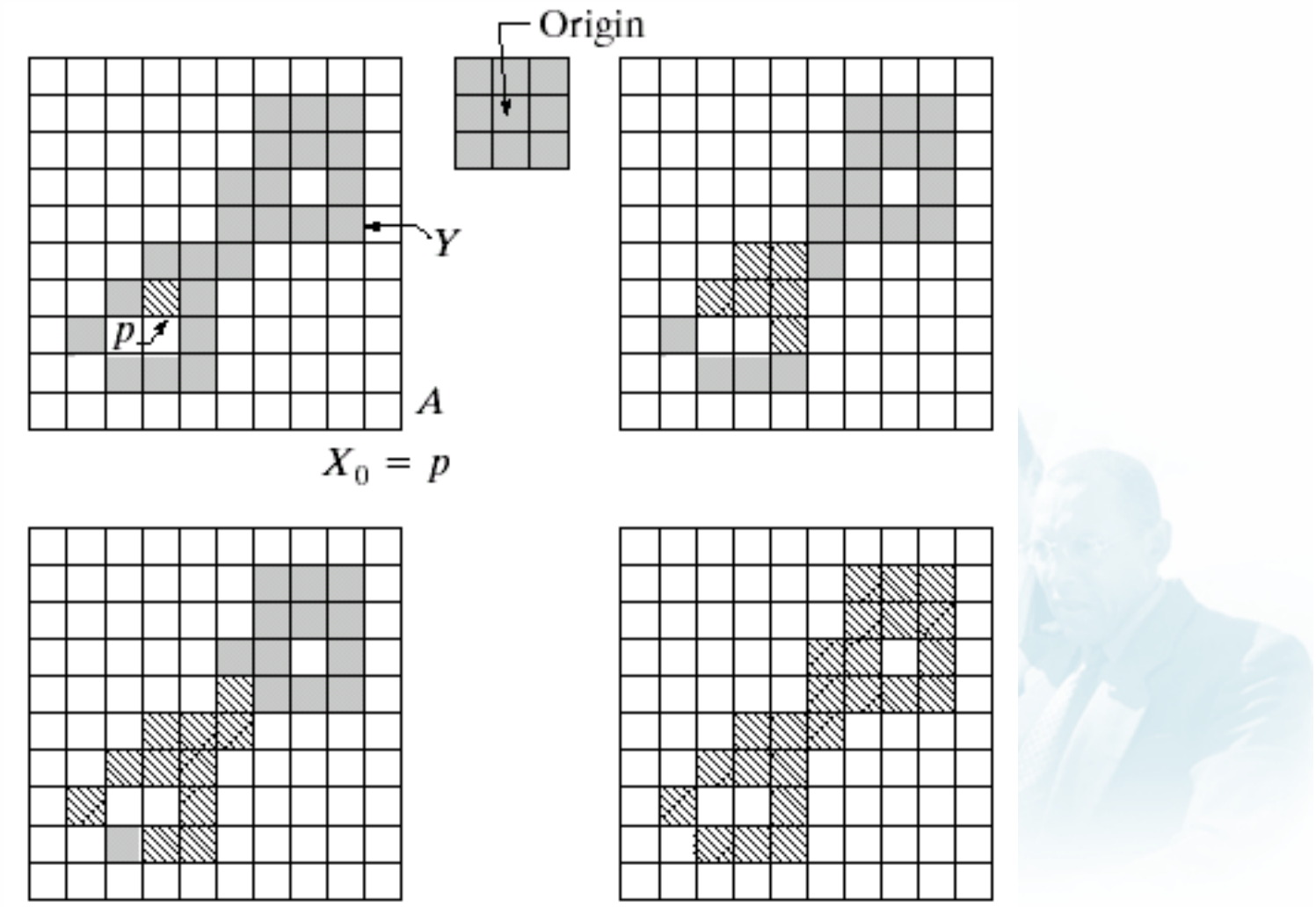
- Idea: start from a point in the connected component, and dilate it iteratively

$$X_0 = p$$

$$X_k = (X_{k-1} \oplus B) \cap A, \quad k = 1,2,3,...$$

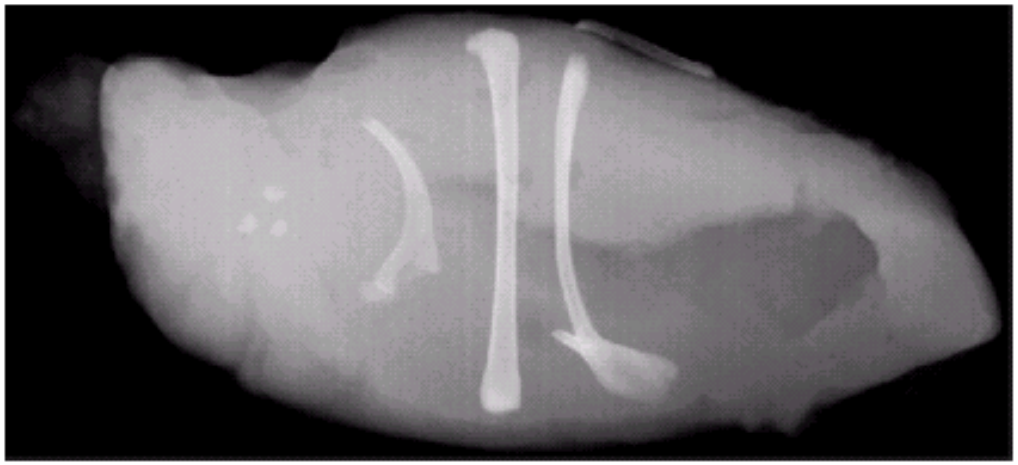
Until $X_k = X_{k-1}$

Extraction of connected components (cont.)



original

雞肉



thresholding

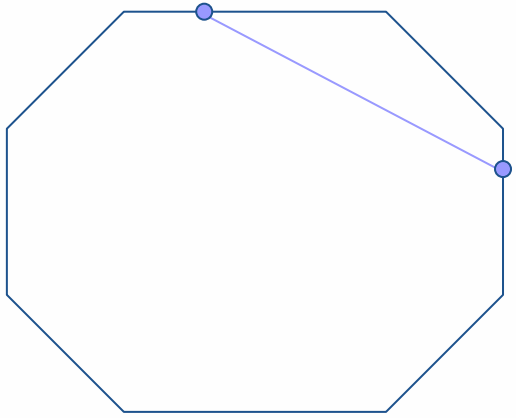
erosion

去除小雜訊

Connected component	No. of pixels in connected comp
01	11
02	9
03	9
04	39
05	133
06	1
07	1
08	743
09	7
10	11
11	11
12	9
13	9
14	674
15	85

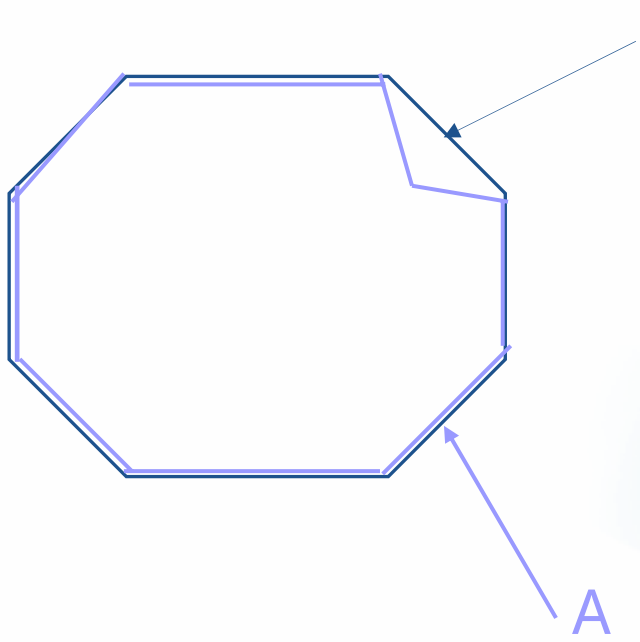
Convex hull

- A set is convex if
  - A line join two points in A lies entirely within A



Convex hull (cont.)

- Convex hull of an arbitrary set A
  - The smallest convex set containing A



Convex hull algorithm

- Idea: find points within line segments

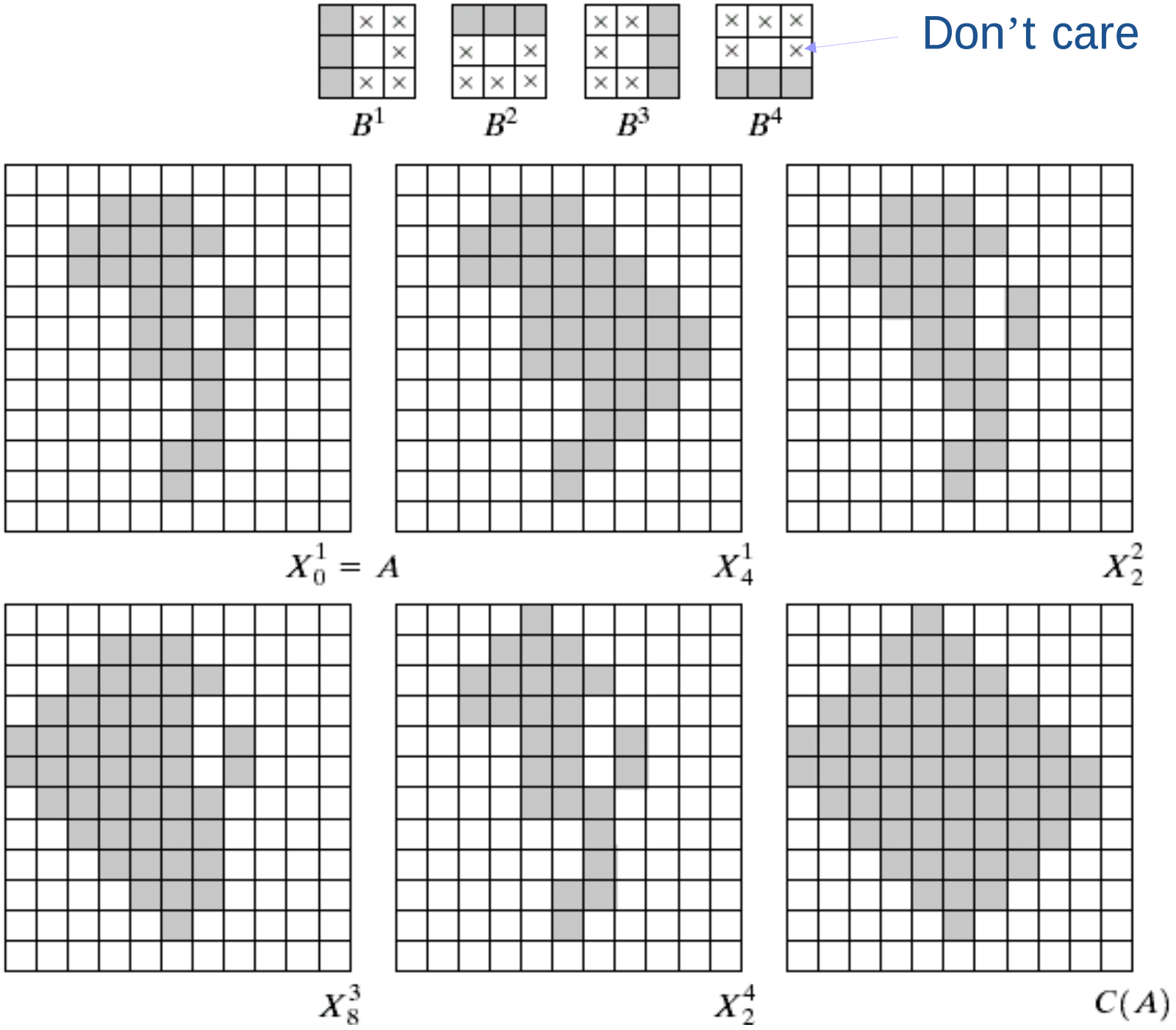
$$X_0 = A \qquad B^i : \text{structuring elements}$$

$$X_k^i = (X_{k-1}^i \circledast B^i) \cup A, \quad k = 1,2,3,\ldots$$

Until $X_k^i = X_{k-1}^i$

Hit-or-miss trans., no background match is required = erosion?

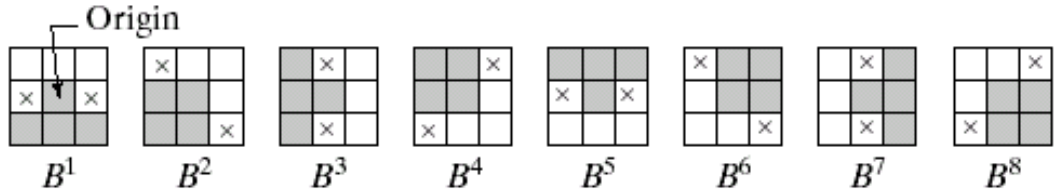
$$C(A) = \bigcup_i X_{conv}^i$$

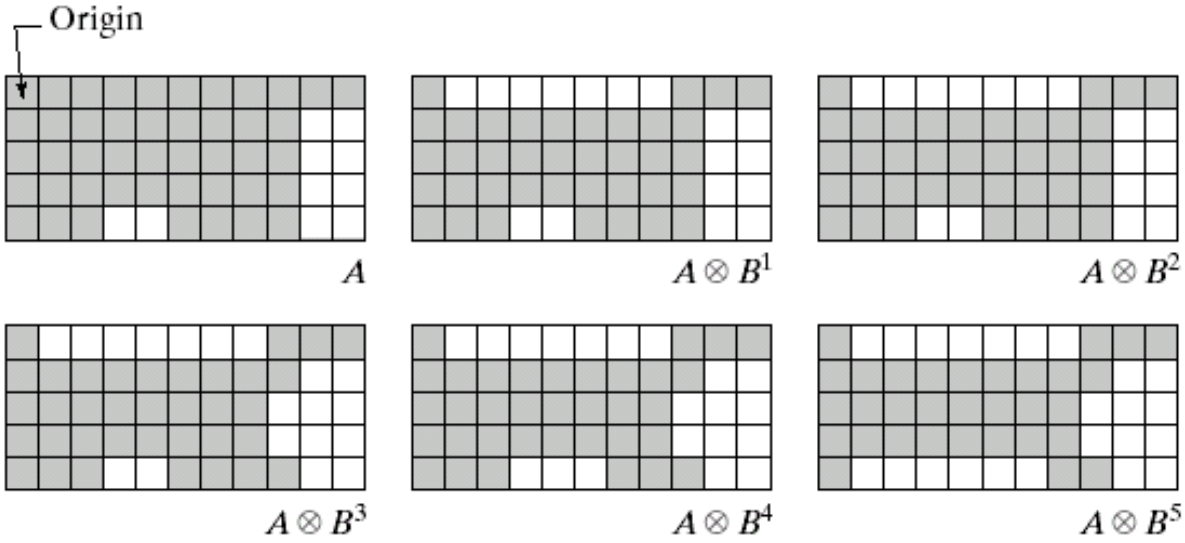

Don’t care
B^1
B^2
B^3
B^4
$X_0^1 = A$
X_4^1
X_2^2
X_8^3
X_2^4
$C(A)$

Thinning 細化

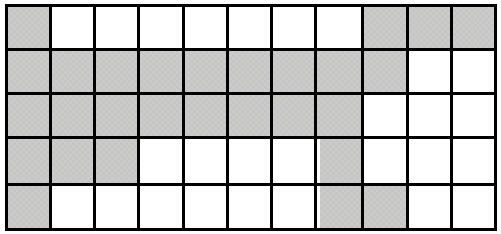
$$A \otimes B = A - (A * B)$$

$$\{B\} = \{B^1, B^2, B^3, ..., B^n\}$$

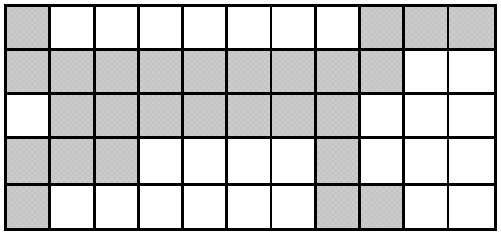




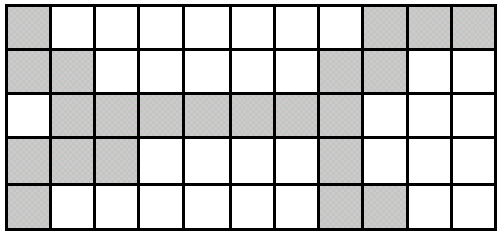
Thinning (cont.)



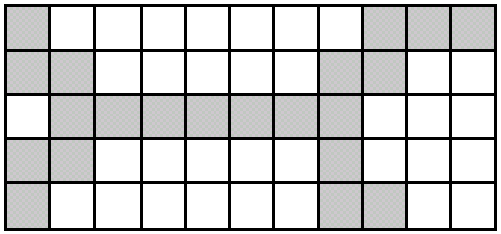
$A \otimes B^{6}$



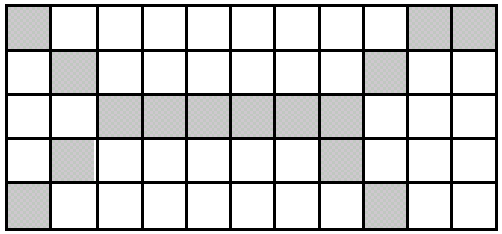
$A \otimes B^{7,8}$



$A \otimes B^{1,2,3}$



$A \otimes B^{4,5,6,7,8,1,2,3}$

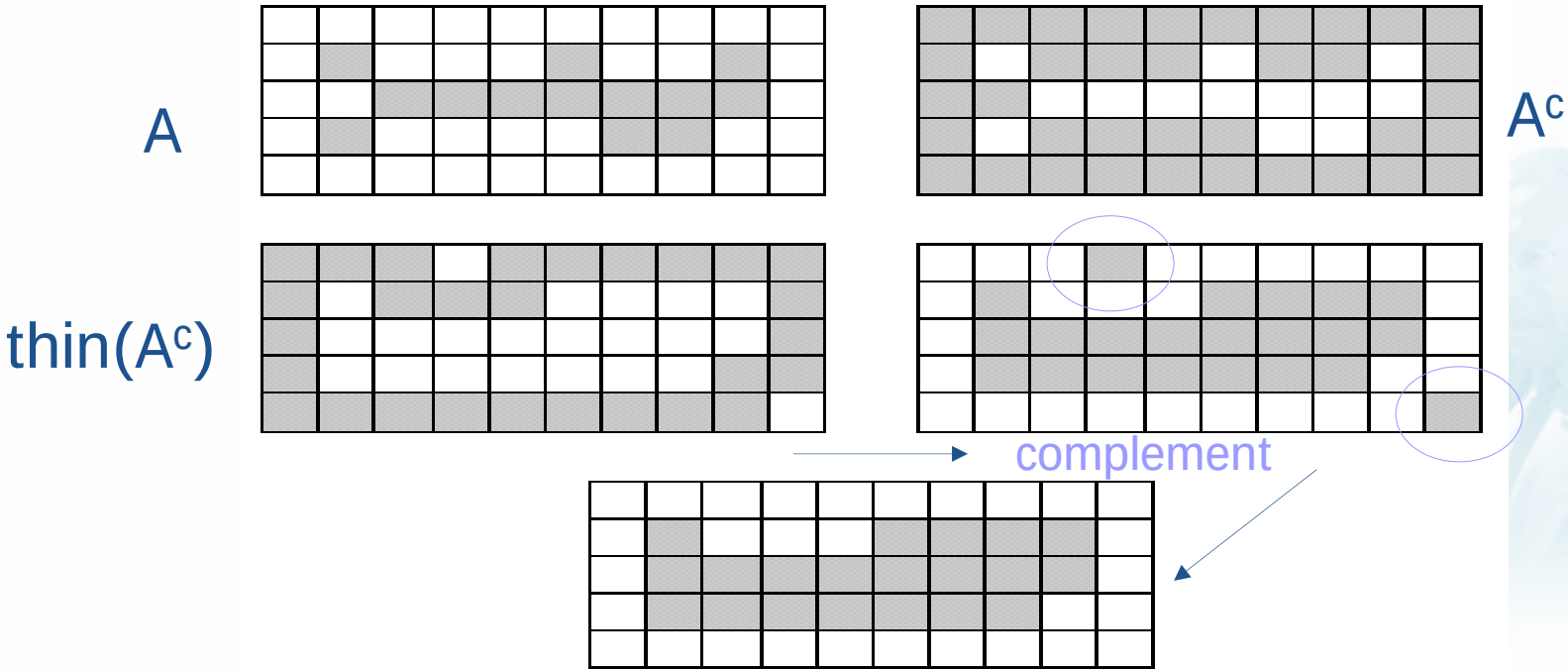


Convert to m-connectivity

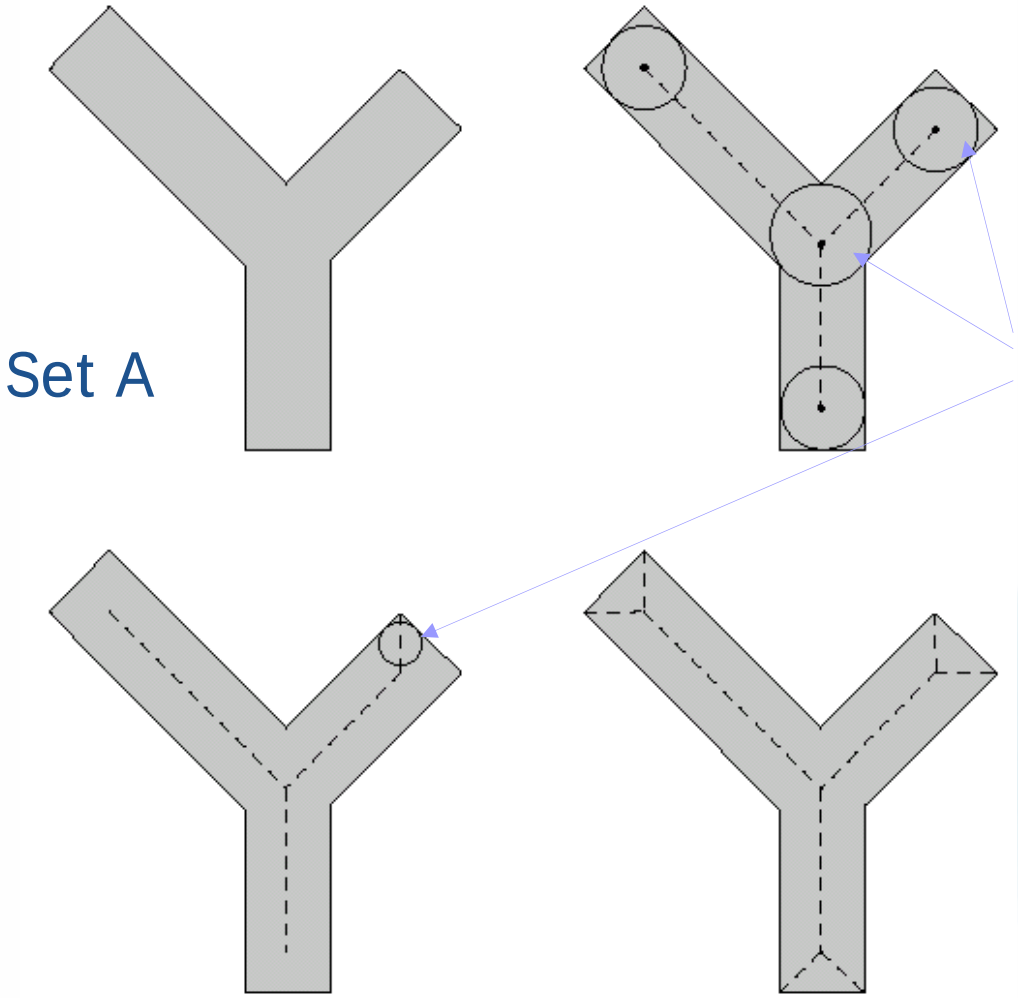
Thickening

$$A \odot B = A \cup (A \circledast B)$$

Another method: 1. Thin the background
2. Complement the result



Skeletons 骨架



Maximum disk

1. The largest disk Centered at a pixel
2. Touch the boundary of A at two or more places

Skeletons : algorithm

- Idea: 不斷的 erosion

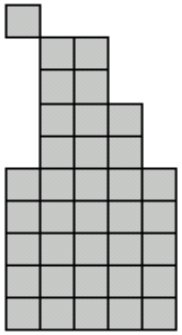
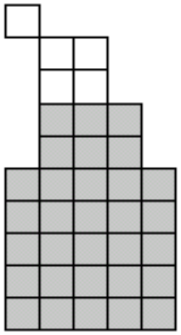
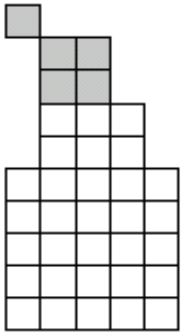
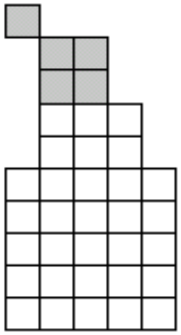
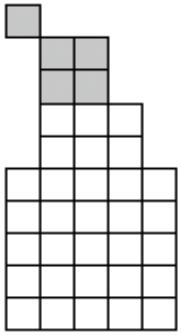
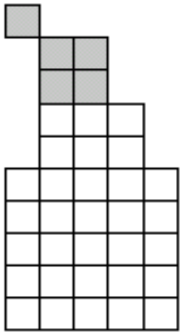
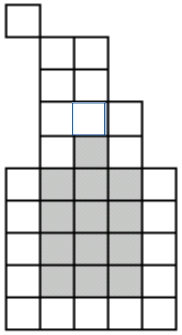
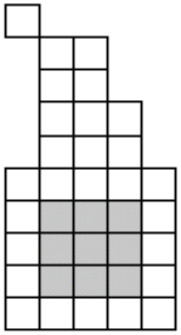
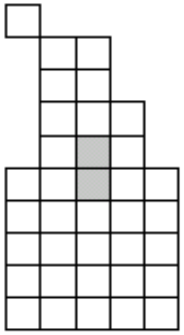
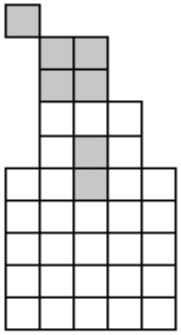
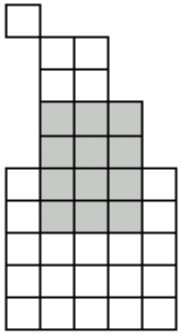
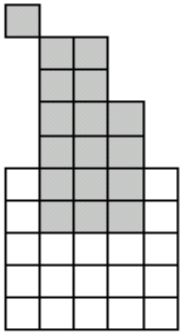
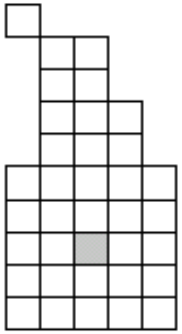
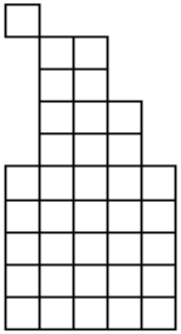
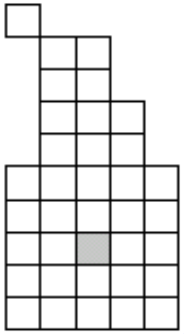
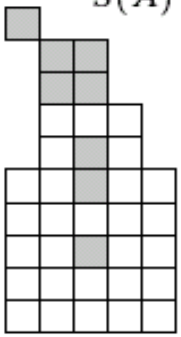
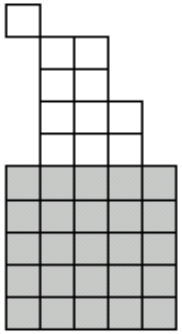
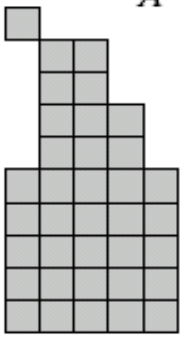
$$S(A) = \bigcup_{k=0}^{K} S_k(A)$$

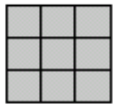
$$S_k(A) = (A \ominus kB) - (A \ominus kB) \circ B$$

$$(A \ominus kB) = (\ldots(A \ominus B) \ominus B) \ominus \ldots) \ominus B$$

Erosion k 次

$$K = \max\{\ k \ | (A \ominus kB) \neq \varnothing\ \}$$

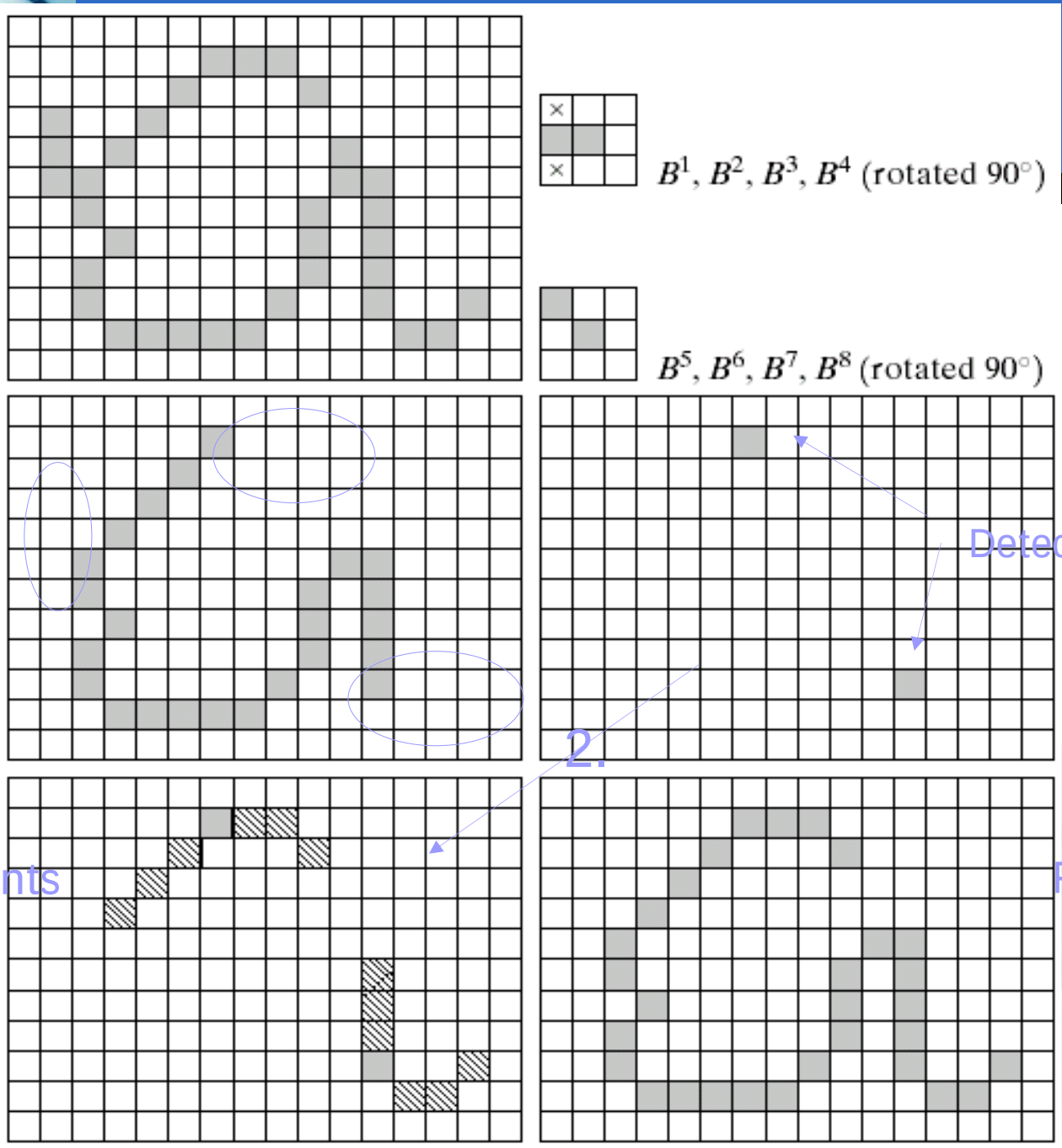
k	$A \ominus kB$	$(A \ominus kB) \circ B$	$S_k(A)$	$\bigcup_{k=0}^{K} S_k(A)$	$S_k(A) \oplus kB$	$\bigcup_{k=0}^{K} S_k(A) \oplus kB$
0						
1						
2				$S(A)$ 		A 

B 

Result is not connected!

Pruning

- Clean up the parasitic components of the set
  - Complement step of thinning and skeletons
- Application: hand-printed character recognition
  - Find skeleton of each character
  - Produce spurs during erosion by non-uniformities in the strokes



1. Detect end point iteratively

$$X = A \otimes B$$
$$= A - (A \circledast B)$$

thinning

Hit-or-miss

Detect endpoints

2.

2. Dilate endpoints

Final result

Structuring elements

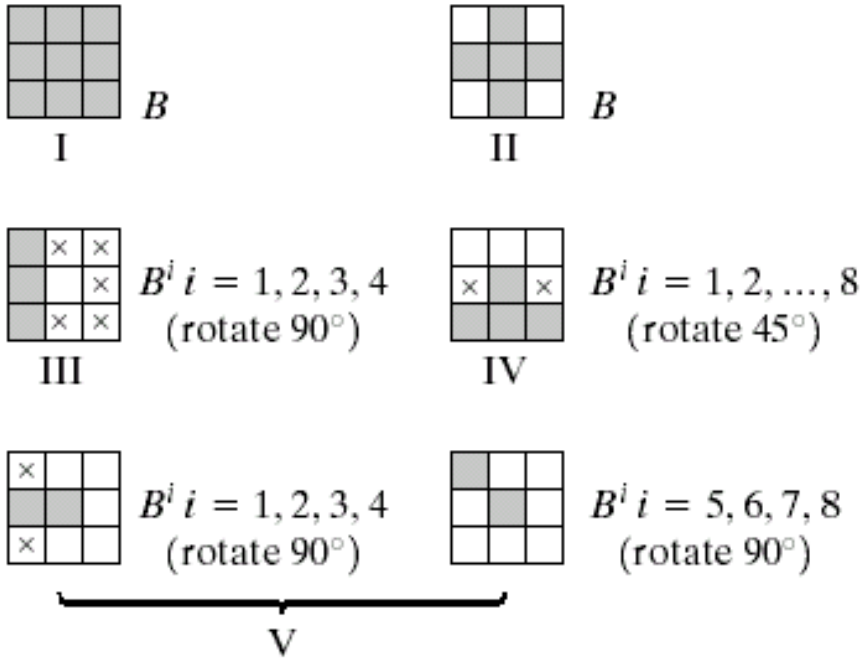


FIGURE 9.26 Five basic types of structuring elements used for binary morphology. The origin of each element is at its center and the ×'s indicate "don't care" values.

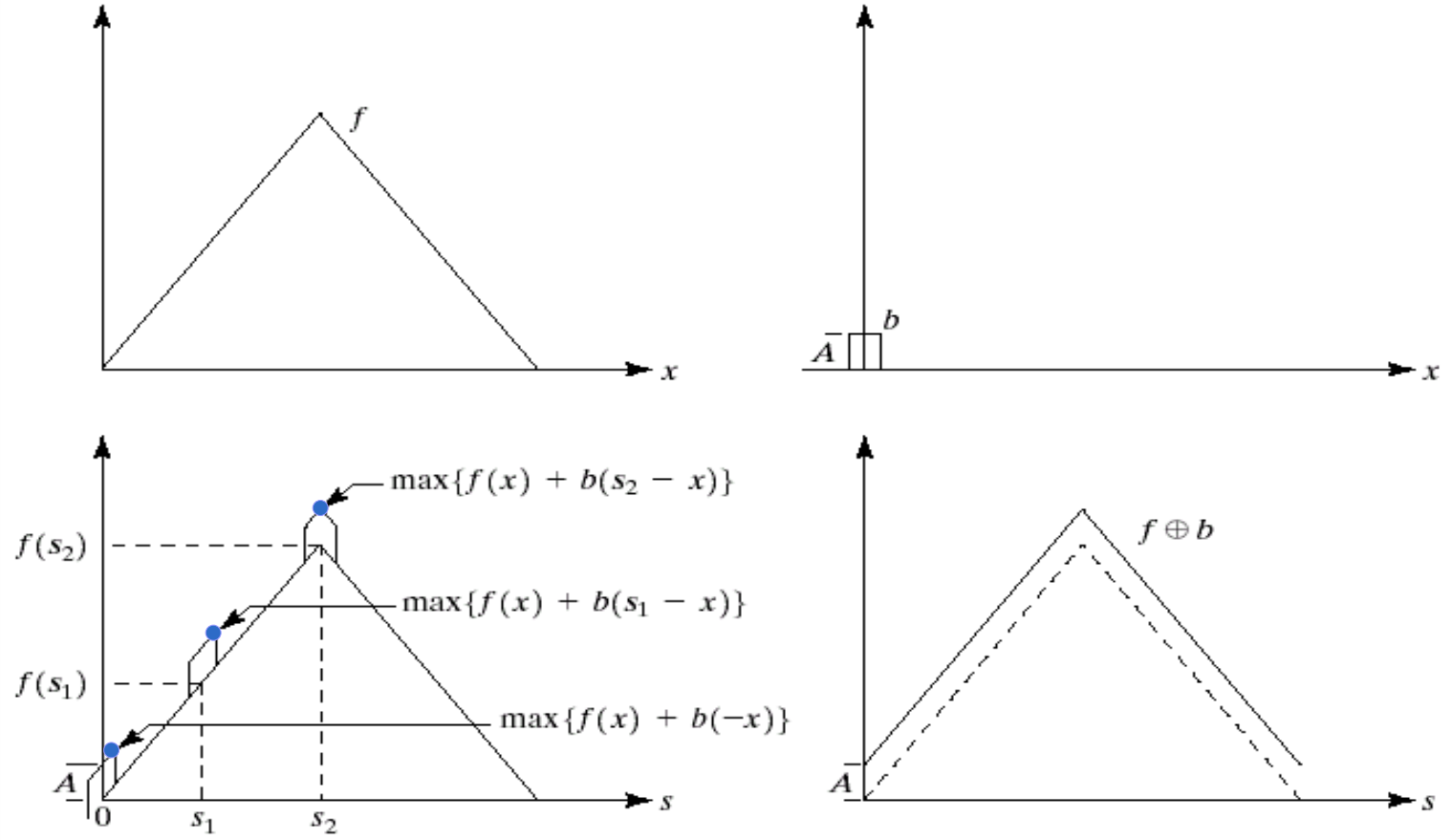
Morphological operations applied to gray-level images

- $f(x,y) \in Z$: input image
- $b(x,y) \in Z$: structuring element

Dilation

convolution-like operation

$$(f \oplus b)(s,t) = \max\{f(s-x,t-y)+b(x,y) \mid (s-x,t-y) \in D_f ; (x,y) \in D_b\}$$



Dilation (cont.)

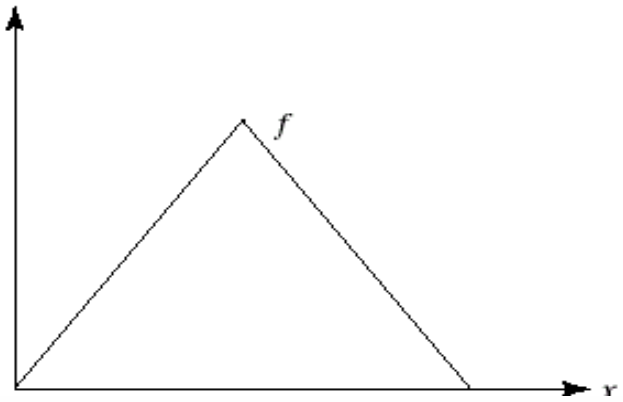
1. Image appears brighter
2. Size of dark regions become smaller

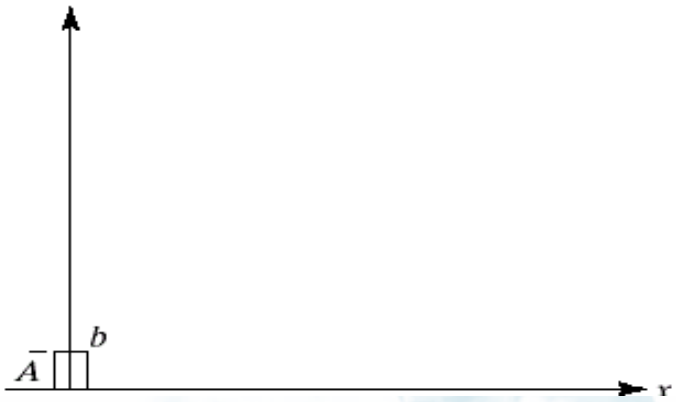
original

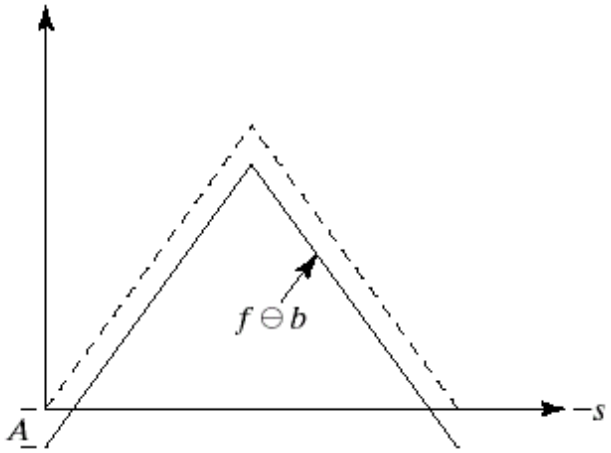
Dilated with 5x5 mask

Erosion

$$(f \ominus b)(s,t) = \min\{f(s+x,t+y) - b(x,y) \mid (s+x,t+y) \in D_f; (x,y) \in D_b\}$$



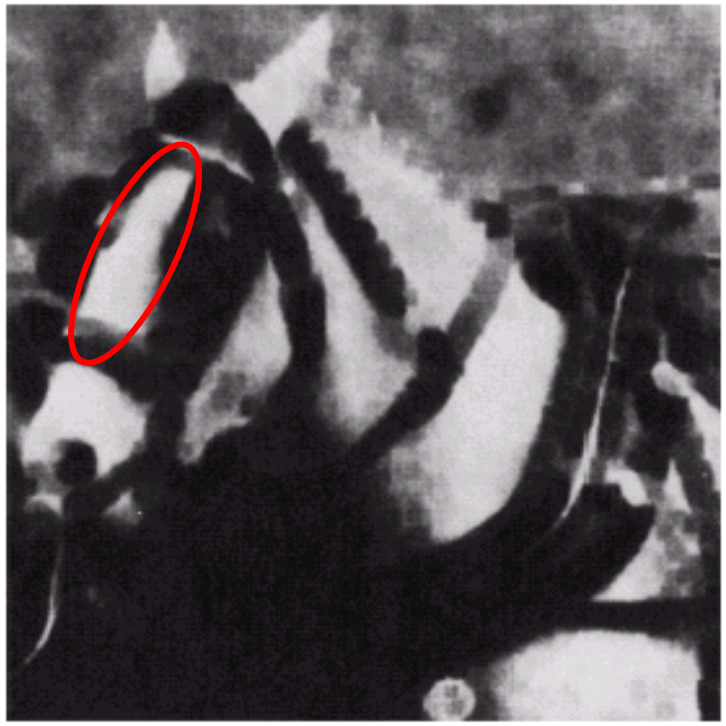




Erosion (cont.)

1. Image appears darker
2. Size of bright regions become smaller

original



eroded with 5x5 mask

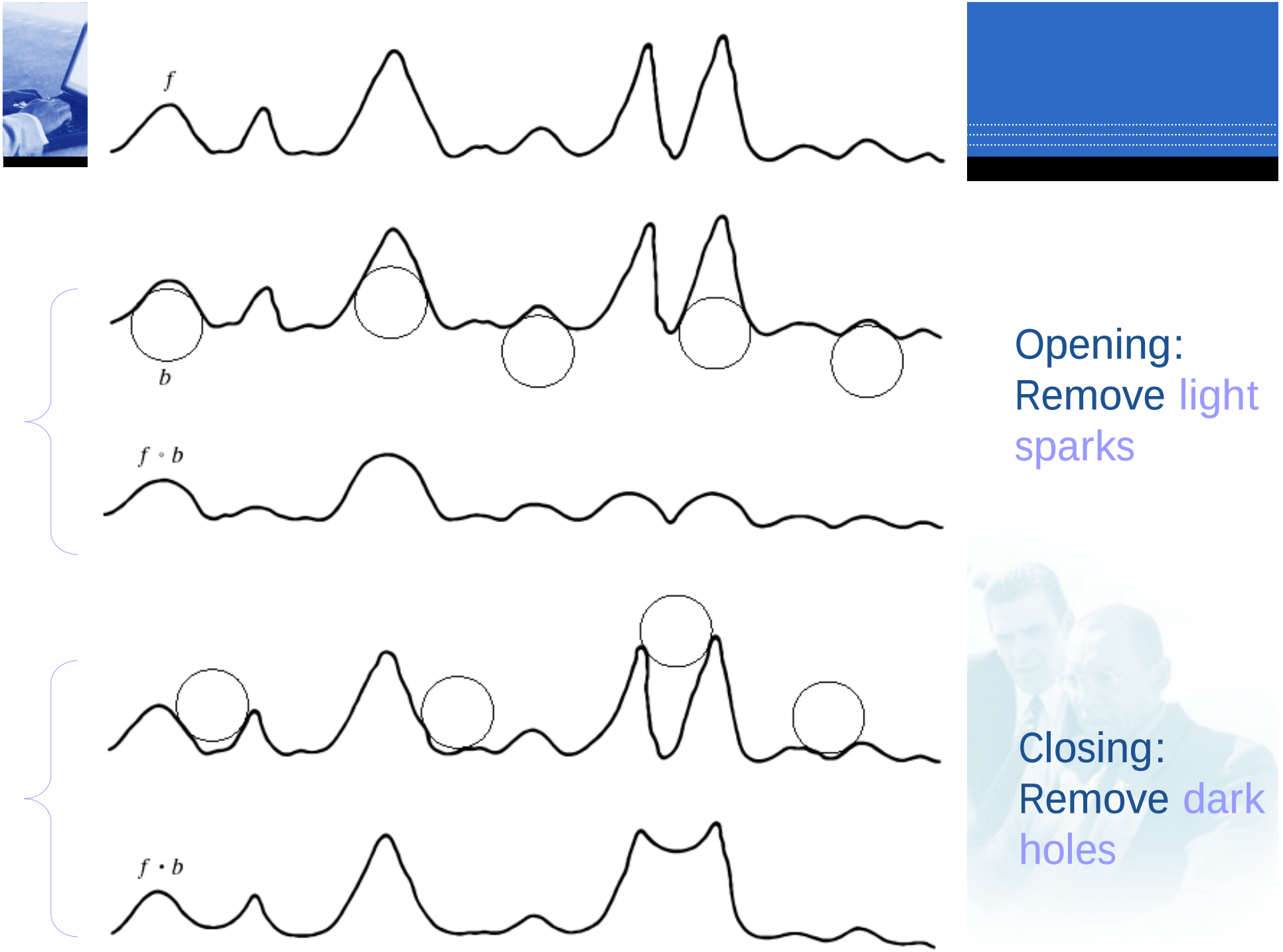
Opening and closing

- Opening = erosion followed by dilation

$$f \circ b = (f \ominus b) \oplus b$$

- Closing = dilation followed by erosion

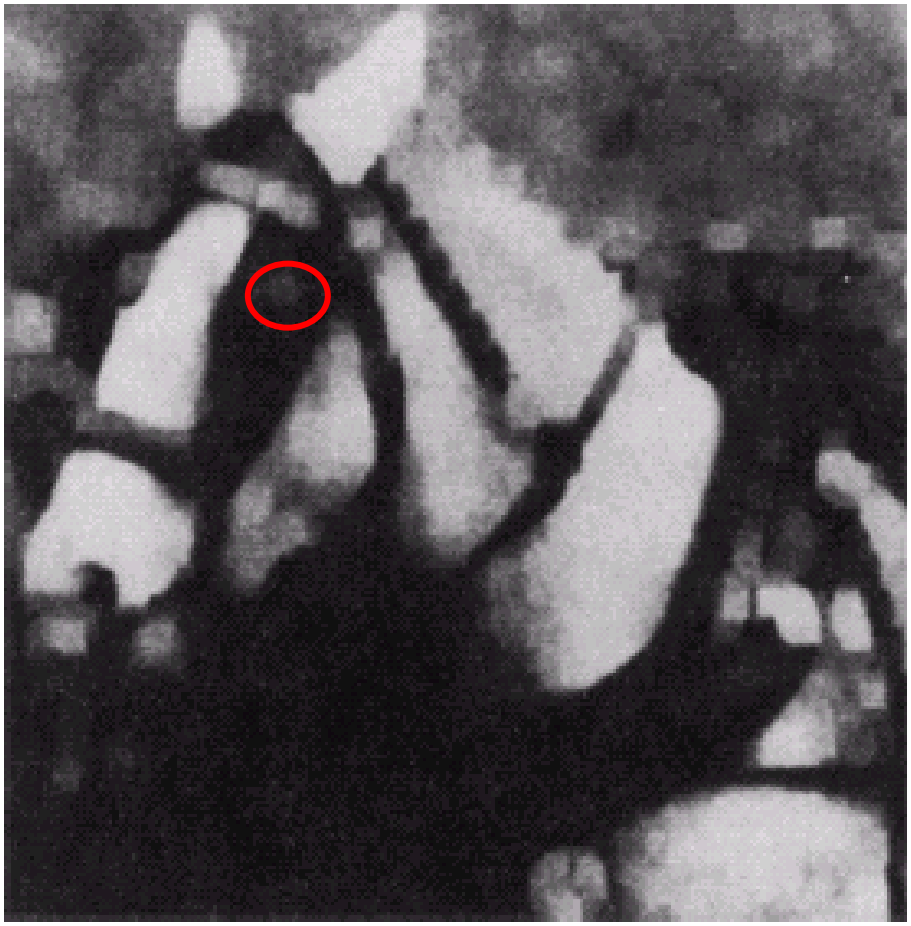
$$f \bullet b = (f \oplus b) \ominus b$$



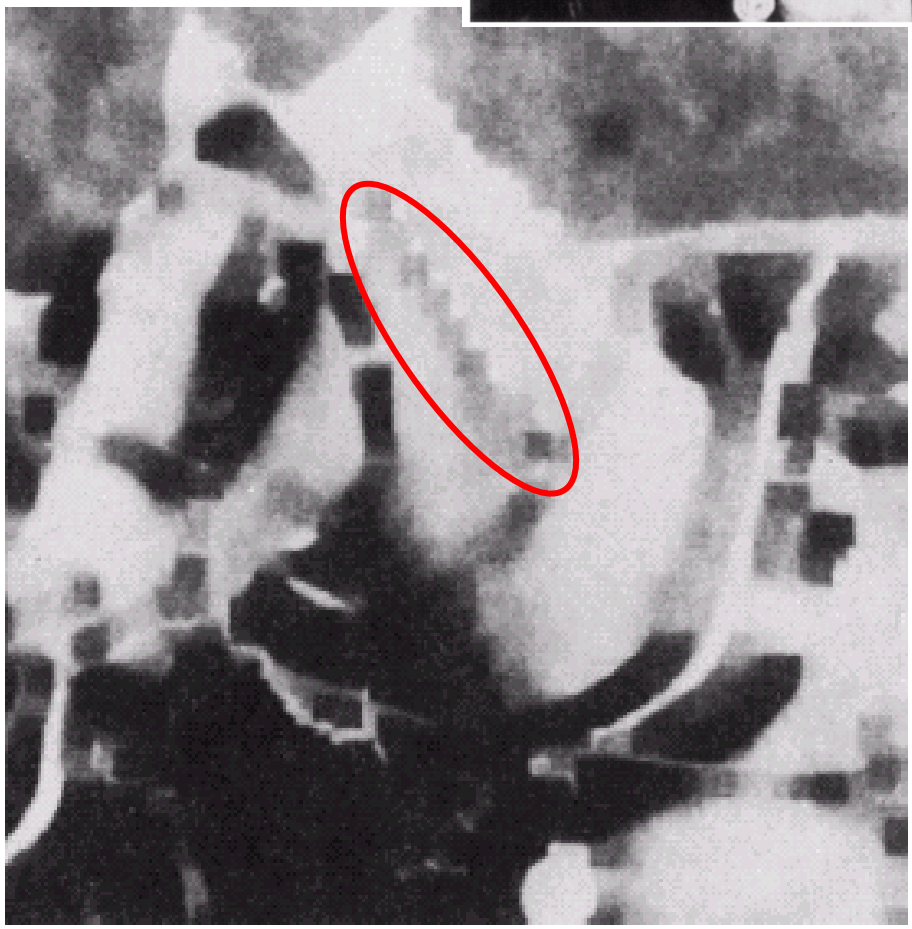
Opening:
Remove light sparks

Closing:
Remove dark holes

Example of opening and clos



opening



closing

Morphological smoothing

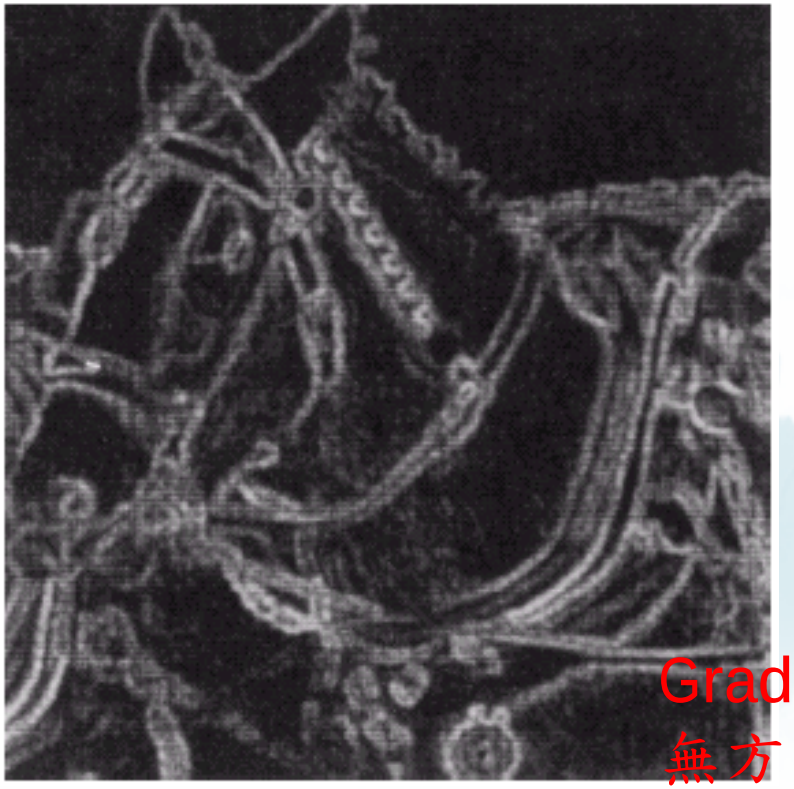
- Opening+closing
  - Remove both light and dark details

Morphological gradient

- Highlight sharp gray-level transition

$$g = (f \oplus b) - (f \ominus b)$$

original

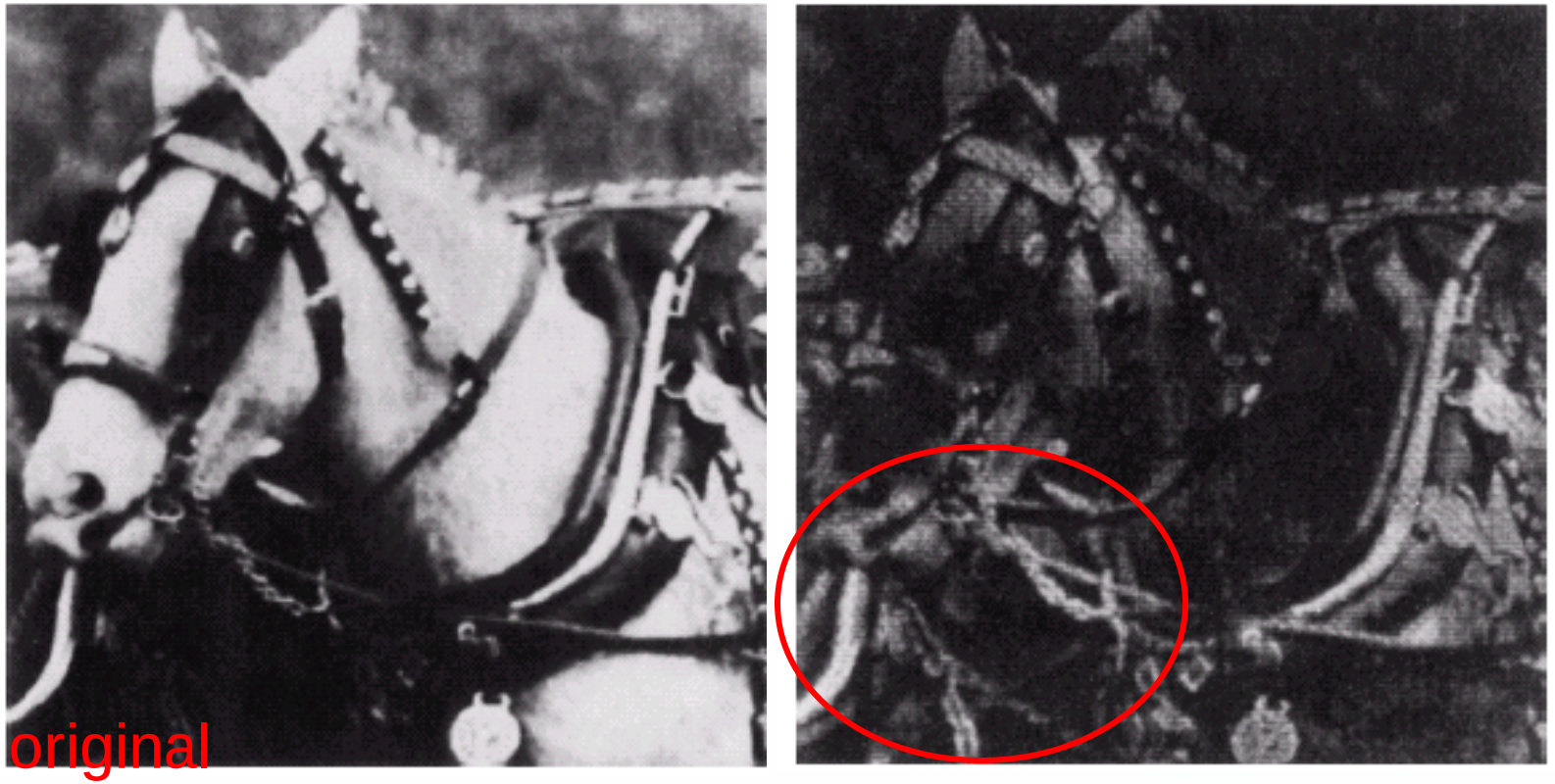


Gradient:
無方向性

Top-hat transformation

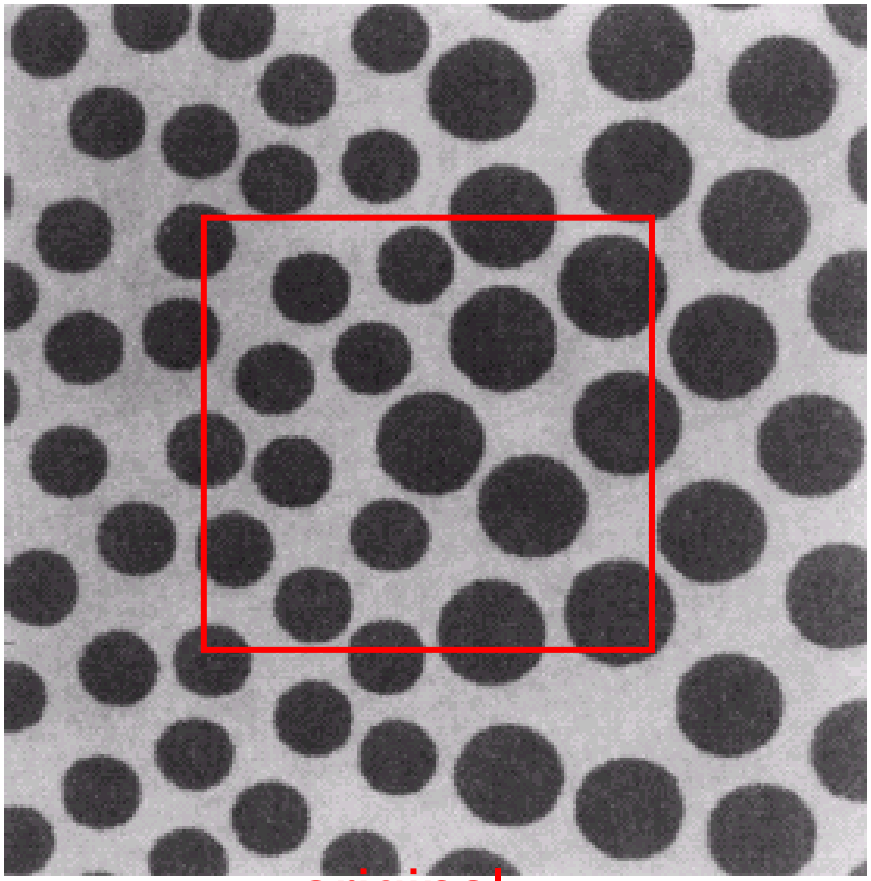
- Enhance detail in the presence of shading

$$h = f - (f \circ b)$$

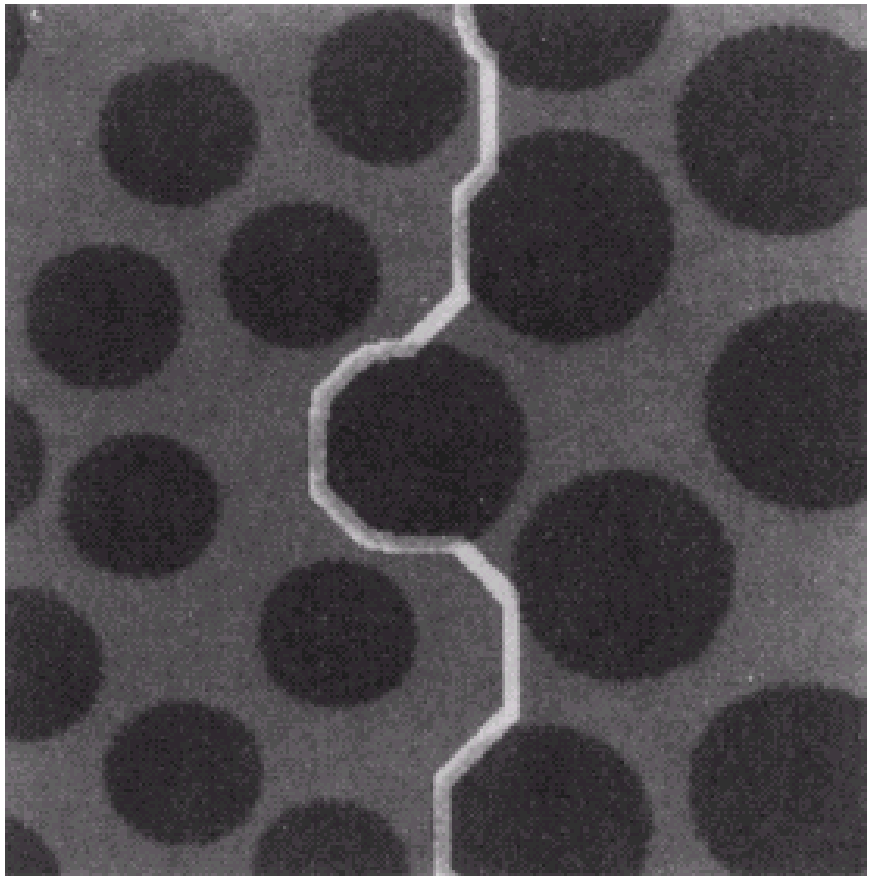


Texture segmentation

Closing with circles of different sizes to identify dark circles



original



Granulometry

Opening with circles of different sizes to identify light circles